
Course Outcomes

Note: Book Chapters that correspond to the listed course outcome(s) are provided in square brackets [].

Introduction to Transport Processes

Transport Phenomena refers to the study of the motion and balance of momentum, heat, and mass in engineering problems. These three modes of transport are studied concurrently for several reasons: they have similar molecular origins, they yield similar governing equations/principles, they often occur simultaneously, and they require similar mathematical/conceptual tools.

In this section we define and introduce several conceptual tools necessary for studying transport, and answer several pertinent engineering questions:

What are my options in visualizing/conceptualizing the movement of momentum, heat, mass?

- Discuss the relationship between Thermodynamics and Transport Processes
- Summarize the critical aspects of continuum mechanics
 - Explain the continuum hypothesis and the origin of its breakdown [Ch 1.1, 1.2]
 - Define and give examples of (fluid) property fields [Ch 1.3]
- Differentiate between an Eulerian and Lagrangian description
 - Explain the difference between an Eulerian and Lagrangian viewpoints [Ch 3.2]
 - Identify and differentiate between streaklines, streamlines, and pathlines [Ch 3.3, 3.4]
 - Mathematically derive streaklines, streamlines, and pathlines from an Eulerian velocity field
 - Define systems and control volumes and identify when each is a useful frame of reference [Ch 3.5]

By what underlying mechanisms does this transport take place? [Ch 3.1]

- Explain and give examples of the three primary modes of heat transfer [Ch 15.1-15.4]
- Describe the primary modes of mass transfer [Ch 24.1-24.3]
- Identify the underlying forces and conceptual hurdles in momentum transport
 - Name and explain the origin of forces acting on fluids in a control volume [Ch 2.1-2.4]
 - Define laminar, turbulent, and transition flow regimes [Ch 12.1]
 - Distinguish between external and internal flows
 - Explain the meaning of fully-developed flow and calculate the entrance region length necessary for a pipe flow

What global understanding of the problem can be achieved through simple reasoning?

- Apply dimensional analysis to generalize problem descriptions
 - Perform simple dimensional analysis, using the Buckingham Pi method [Ch 11.1-11.4]
 - Calculate the Reynolds Number and use it to predict flow regimes

Linear Transport Relations

Much of Transport Phenomena deals with the *exchange* of momentum, mass, or heat between two (or many) objects. Often, the most mathematically simple way to consider how and how fast exchanges take place is to look at *driving forces* and *resistances*.

In momentum transport, we are interested in driving forces that arise from differences in pressure and/or velocity.

- Solve problems in fluid hydrostatics
 - Derive the pressure field equation [2.1]
 - Calculate the pressure distribution in a fluid or system of fluids that is at rest [2.2]
 - Use Archimedes' principle to calculate buoyant forces on (partially) immersed objects [2.3-2.4]
- Use friction factors and/or drag coefficients to calculate viscous (and pressure) forces in fluids [12.2, 13.2-13.4]
 - Distinguish between lift, drag, skin friction, and form drag
 - Calculate friction factors from correlations and read friction factors off of charts
 - Use drag coefficient correlations and/or charts to calculate drag on submerged objects (external flows)
 - Estimate frictional energy losses in pipes and pipe networks

In heat and mass transport, our driving forces arise from differences in concentration and temperature.

- Perform convection and convection/radiation problems
 - Perform convective heat transfer calculations [15.3, 19.1, 19.2]
 - Perform convective mass transfer calculations [24.3, 28.1, 28.2]
 - Perform radiative heat transfer calculations [15.4, 23.1, 23.2, (23.7)]
 - Calculate the thermal resistance and magnitude of heat flow in combined convective/radiative systems [15.5]

Constitutive Laws

The mathematical analysis of the diffusion of heat, mass, or momentum is incorporated into *constitutive laws* that relate this diffusion to easily measurable quantities (like temperature, velocity (pressure), and concentration).

- Use a resistor analogy to solve for heat flows [15.5, 17.1]
 - Calculate the thermal resistance and magnitude of conductive heat flow/flux through a planar wall
 - Calculate the thermal resistance and magnitude of conductive heat flow/flux through multiple planar walls
 - Calculate the thermal resistance and magnitude of conductive heat flow/flux through a cylindrical shell
 - Calculate the thermal resistance and magnitude of conductive heat flow/flux through a spherical shell
 - Calculate the resistance and magnitude of heat flow in systems in which multiple modes of heat transfer are present
 - Extend the resistor analogy to non-one-dimensional problems using shape factors [Ch 17.4]
 - Determine the heat flow through these solids from their temperature profiles
 - Use film theory and other correlations to obtain h [19.5]
- Explain and calculate viscous stresses [7.1, 7.2, 7.4]
 - Calculate viscous stresses/forces from velocity distributions

- Identify Newtonian and non-Newtonian behavior from stress versus strain curves
- Solve diffusive mass flows problems
 - Explain the difference between the total flux and the diffusive flux [24.1]
 - Calculate the magnitude of diffusive mass flow/flux through a planar film in equimolar counter-diffusion [25.4]
 - Calculate the magnitude of diffusive mass flow/flux through a planar *stagnant* film [26.1]
 - Calculate the magnitude of diffusive mass flow/flux for systems with non-zero bulk flow
 - Calculate the magnitude of diffusive mass flow/flux through a cylindrical and spherical shells
 - Use film theory and other correlations to obtain k_c [28.6]
 - Use the two-resistance model to perform fluid-fluid mass transfer calculations [29.1-29.3]
 - Calculate mass/molar flows/fluxes from concentration profiles

A *microscopic* or *continuum* description of transport requires that we examine "diffusion" of our conserved quantities at the molecular level.

- Estimate transport properties from molecular calculations
 - Explain the molecular origins of fluid viscosity and shear stresses.
 - Estimate fluid viscosities [7.3]
 - Explain the molecular origins of thermal and mass diffusion/conduction [15.2, 24.2]
 - Estimate thermal and mass diffusivities/conductivities [15.2, 24.2]

Analyzing Mass and Heat Transfer Equipment

Scaling up to solving problems using process equipment requires both continuum and macroscopic knowledge of transport, and is industrially quite significant.

- Analyze heat transfer equipment
 - Distinguish between exchanger types [22.1]
 - Solve the "thermodynamic" problem [22.2]
 - Visualize the changing driving forces in co-current versus counter-current exchangers [22.2]
 - Calculate the "average" driving force from the heat flow and resistance [22.2]
 - Calculate the heat duty of an exchanger from the "average" driving force and total resistance [22.2]
 - Calculate the heat flow in an exchanger from the exchanger's effectiveness [22.4]
- Analyze mass transfer equipment [31.5, 31.6]
 - Calculate the mass exchanged in a well-mixed contactor or the time necessary to achieve a particular exchange
 - Calculate the "average" driving force in a continuous contactor
 - Calculate the mass exchange in a continuous contactor and/or the size of the column

Elementary Non-Steady Phenomena

Because Transport deals with *rates* it is often the case that we must consider non-steady (or transient) operation (when the rates do not exactly cancel). In this section, we examine *scalar* transport (heat and mass transfer) where some non-steady problems can be simplified significantly.

- Explain the utility of the Biot number [18.1]

- Identify "regimes" of transient response based on the value of the Biot number
- Use the "lumped" equation to solve "1D" transient heat transfer problems [18.1]
- Use the "lumped" equation to solve "1D" transient mass transfer problems [27.1]
- Use Gurney-Lurie charts to solve heat and mass transfer problems [18.2, 27.4]
- Use the semi-infinite approximation to solve both transient heat and mass problems for "short" times [18.1, 27.2]

Shell/Integral Balances

Shell or integral (macroscopic) balances are often relatively simple to solve, both conceptually and mechanically, as only limited data is necessary. At the same time, many problems require only the level of detail that may be extracted from these types of balances and thus they represent powerful tools.

- Perform control volume (or macroscopic) balances
 - Derive and use a macroscopic mass balance (continuity equation) [4.1, 4.2]
 - Derive and use a macroscopic momentum balance [5.1, 5.2]
 - Derive and use a macroscopic energy balance [6.1, 6.2]
- Use the General Mechanical Energy Balance Equation to perform calculations on simple pipe networks and/or process problems [6.3]
 - Explain the connection between a macroscopic energy balance and Bernoulli's Equation
 - Use Bernoulli's Equation to calculate velocities, pressures, or height changes in flowing fluid systems
 - Extend the Bernoulli's Equation for more general cases (i.e., write and use the mechanical form of the macroscopic energy balance equation)
- Solve heat transfer problems on macroscopic bodies [p241-243]
 - Determine the efficiency of a fin
 - Calculate the heat flow through a fin from the fin's efficiency

Differential Balance Equations

Differential balances, although more complex to solve, can yield a tremendous wealth of information about ChE processes. General balance equations for each of the modes of transport can easily be derived either directly from shell balances or via control volume analysis. Understanding the origin and meaning of the terms that make up these balance equations lies at the heart of posing and solving complex transport problems.

- Describe the physical meaning of each of the terms within a general thermal energy balance
 - Simplify the general thermal energy balance equation (identifying assumptions)
 - Identify reasonable boundary conditions in a conduction problem (explain when each is most useful)
- Describe the physical meaning of each of the terms within a differential mass balance equation
 - Simplify the mass balance equation (identifying assumptions)
 - Identify reasonable boundary conditions in a mass transport problem (explain when each is most useful)
- Describe the physical meaning of each of the terms within the momentum balance
 - Explain the physical meaning of the terms in the differential Continuity and Navier-Stokes Equations

- Simplify both the differential Continuity and Navier-Stokes Equations
 - Identify proper boundary conditions for use with the Navier-Stokes equations
-

Introduction to Transport Properties (ITP)

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-

ITP: What is Transport Phenomena?

Transport Phenomena is the study of the transfer of momentum, heat and mass. These three "modes" of transport are usually taught/grouped together because:

- they have similar molecular origins
- they yield similar governing equations/principles
- they often occur simultaneously
- they require similar mathematical/conceptual tools

A few examples of Transport in action may serve to better put this in perspective ...

EXAMPLE:

TWO-PHASE REACTOR *In a two-phase reactor the catalyst is carried in one phase (say, the oil phase) along with one monomer, while the other monomer is carried in the other phase. The reaction will clearly go fastest if we can maximize the surface area between the two phases so that the "contact" of the reactants is better.*

What sort of device would be best suited for this task? How would you run the device?

Perhaps you would run this in a tank with a bladed impeller. This would allow you to prevent the fluids from separating via gravity and also to create small droplets. The small droplets would be useful from a mass transfer perspective because there would be more contact area between the phases.

What properties of the fluid and device will be most important? What operating conditions?

The two fluid densities would determine how rapidly they would settle. The viscosities would determine the power required to move the impeller (and the impeller speed would dictate the mixing rate and droplet size). The conductivity of the fluids would determine what the cooling requirements might be if this is an exothermic reaction.

EXAMPLE:

UREASE *A person undergoing kidney failure needs a device to aid them until a transplant become available. You decide to encapsulate the urea-destroying enzyme urease in a solid pellet and build a "reactor" (artificial organ) to do the task.*

What size of pellet should be used?

A smaller pellet would allow better contact between the bulk fluid (blood) and the pellet reducing mass transfer limitations; however, small particles will also make it more difficult to "push" the fluid past

the particles (ever try to pack flour into a straw and then suck water through it?!).

What material properties of the pellet will impact its effectiveness? As in the previous example, we need to worry about heat transfer either to or from the pellet (center) depending on whether the reaction requires or releases heat. Other than that, it would be useful if the pellet was porous so that we could put more catalyst on the "surface", but we need to worry about how fast mass transfer will occur through those pores.

What should the flow-field around the pellet look like?

Generally, the faster the fluid flow past the pellet, the better the heat and mass transfer to/from the pellet will be; however, as with small pellets, fast flow might require too much (mechanical) energy, that is, too "big" of a pump.

EXAMPLE:

HEATER *You wish to design a better home heater to offset the rising prices of oil. You plan on designing a solar collector to go on the roof of your house and harness the energy of the sun.*

How will Pittsburgh's weather impact your design?

The most obvious thing is that cloudy days will absorb much of the radiant energy from reaching the collectors. Also, the efficiency of the collectors may depend on temperature so that you may need to devise a way of keeping them warm.

Once you "capture" the heat how will you move it into the house?

The traditional way is to heat up a fluid like air or water and simply flow that through the house. Air is nice because it requires little energy to move, but it also has a very low heat capacity (so it cannot carry much heat). Water on the other hand is much more viscous, but it can also carry much more heat and has a better conductivity so you can move heat in/out of water more rapidly.

ITP: Relation to Thermodynamics

The subject of this course is Transport Phenomena. In particular, we will examine the transfer of momentum, heat and mass. While Thermodynamics tells us that heat and/or mass *will* be transferred as a system moves from one equilibrium state to another, it does not tell us *how* or *how rapidly* this transfer will take place. (because this transfer is inherently a non-equilibrium process!)

Nevertheless, there is still a strong connection between the energy balances done in Thermodynamics and Transport...

Energy Balance

There are multiple *forms* of energy that a material may possess:

- **Mechanical** (kinetic or potential)
- Molecular (chemical, electrical, electro-magnetic, or **thermal**)
- Nuclear (we will ignore this one)

These forms may interconvert (i.e., from chemical to thermal), but energy *must be conserved* (if we exclude nuclear reactions). This idea is expressed in Thermodynamics as the First Law: Energy can neither be created nor destroyed.

Conservation expressions will play a central role in this course! One clear difference from Thermodynamics lies in the fact that in Transport we will focus on specific *forms* of energy (thermal, mechanical). A more subtle difference is our focus on **rates** in the conservation equations that are to be used.

Interphase Mass Transfer

We know from Thermodynamics that when two phases are in contact with each other, they will (*eventually*) reach an equilibrium state where the composition of the two phases will be determined (by the temperature, perhaps pressure, and the total composition of the mixture). In Transport Phenomena, we will discuss how *rapidly* the transfer of material will occur from the one phase to the other, and what the composition will be *at different points in space and time* within the phases.

NOTE:

One somewhat tricky thing about the mass transfer part of the class is that we will often assume that the phases are in equilibrium at the interface between them. WHY?!

OUTCOME:

Discuss the relationship between Thermodynamics and Transport Processes

TEST YOURSELF

Determine whether Thermo or Transport can answer the following questions?

- What temperature will a glass of ice-water be when left in the classroom for 1 hour? How long will it stay that temperature?

In the first case we will assume that 1 hour is long enough for the liquid-ice system to reach equilibrium (probably true unless it

is a **huge** glass), so thermo is your ticket and about 0C is your answer. How long it will stay that temperature, however, requires transport, as you need to know how rapidly heat is being sucked out of the surrounding room and into the glass (by convection and/or radiation).

- How many trays/stages should there be in a liquid-liquid extraction column? How big should they be?

*In the second case, you can determine the "ideal" number of stages using only thermo and material balances, as the mass equilibrium at each stages is assumed (note that real trays are not "ideal" **because** of transport not being fast enough for them to always reach equilibrium!). In order to figure out how big the trays need to be, however, you need transport to help you determine what flow pattern you want and what the rate of interphase mass transfer will be.*

ITP: The Continuum Approximation

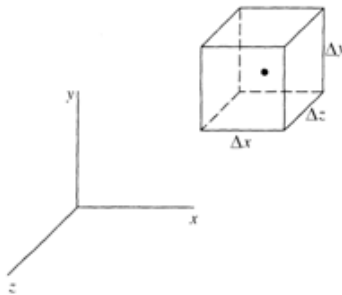
In rigid body mechanics forces act on the center of mass of a body and Newton's Laws of motion are (somewhat) easily applicable. Fluid mechanics, on the other hand, is considerably more difficult as the material deforms *continuously* (i.e., essentially every "piece" of fluid may respond somewhat differently to the force).

DEFINITION:

The **continuum assumption** requires that a fluid is treated as a continuous distribution of matter, or a **continuum**, where properties, velocities, etc. may vary point-by-point.

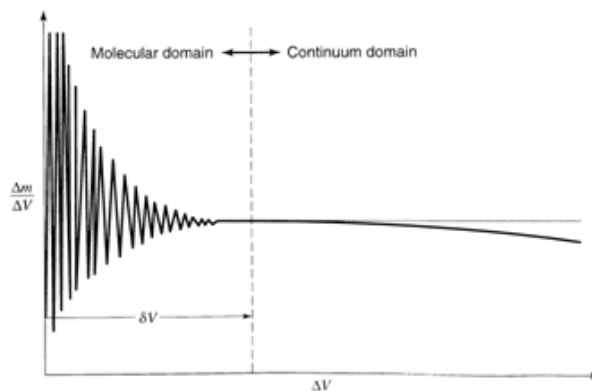
Fluid Density at a Point

In an idealized interpretation of the continuum assumption, the mass (Δm) per unit volume (ΔV) in a fluid, measured at a given point, will tend toward a constant value in the limit as the measuring volume *shrinks* down to zero:



$$\rho = \lim_{\Delta V \rightarrow 0} \frac{\Delta m}{\Delta V} = \frac{dm}{dV}$$

A problem with this requirement is that, as the volume segment gets smaller, you reach a size where random molecular (thermal) motion causes the density to fluctuate, and the continuum approximation begins to break down:



A more accurate requirement, therefore is that the density becomes constant as the volume shrinks to some finite value, δV :

$$\rho(x, y, z) = \frac{dm}{dV} \equiv \lim_{\Delta V \rightarrow \delta V} \frac{\Delta m}{\Delta V}$$

The validity of the continuum assumption then requires that δV has a characteristic dimension several orders of magnitude smaller than the fluid flow domain.

OUTCOME:

Explain the continuum hypothesis and the origin of its breakdown

TEST YOURSELF

Give examples of problems where the continuum approximation breaks down

One obvious answer is the one discussed already: when the system gets to be close to the size of the molecules. In this sense, some nanotechnology problems would be poorly approximated as continua.

*A not-so-obvious example is when you have a mixture of gas and liquid (i.e., a bubbly or frothy system). In some cases, if the scale of your analysis is too close to the scale of the heterogeneities (bubbles, droplets) in the flow, you will need to actually account for **two** continua (the liquid and the gas) separately!*

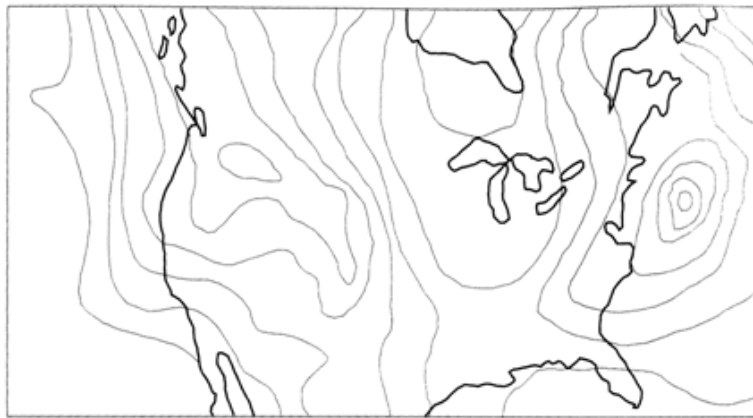
ITP: Property Fields

In general (fluid) properties vary from point-to-point. A property field refers to knowledge of the unique value of a specific property at all points within the fluid:

- Fluid density field (scalar field): $\rho = \rho(x, y, z)$
- Fluid pressure field (scalar field): $P = P(x, y, z)$
- Temperature field (scalar field): $T = T(x, y, z)$
- Concentration field (scalar field): $C_i = C_i(x, y, z)$
- Fluid velocity field (vector field): $\vec{v} = \vec{v}(x, y, z)$

EXAMPLE:

A weather map is a contour plot representation of a 2-D pressure field:



OUTCOME:

Define and give examples of (fluid) property fields

TEST YOURSELF

Give an example and plot a property field

ITP: Streamlines, Streaklines, and Pathlines

Now that we have discussed how one can mathematically describe flow fields (Eulerian versus Lagrangian) as well as reference frames for balances (system versus control volume), it is important to think of visualization of flows. (We already covered (briefly) fluid property fields and showed that for scalar properties, contour plots are sufficient up to 2D).

Three well-accepted flow "visualizers" are the following:

DEFINITION

A **Streamline** is a line that is tangent to the instantaneous velocity field. Imagine "connecting the dots" from a vector field plot of the velocity profile.



DEFINITION

A **Pathline** is a line that is traced out by "watching" the flow of a particular fluid element. Imagine a long-time exposure image of a firefly's motion.

DEFINITION

A **Streakline** is a line that is generated by "tagging" particles that have visited a particular location. Imagine injecting a continuous stream of dye into a flowing fluid.

There are a couple of interesting things to note regarding these lines:

- Since the direction of a streamline is wholly determined by the instantaneous velocity field, and no point may have two different velocities at the same time, *streamlines cannot cross each other* (but they *can* converge to a point, if the velocity goes to zero at that point, which we will call a **stagnation point**)
- Our firefly could easily change his mind and "double-back", so pathlines *can* cross each other
- Similarly, someone could "stir" our fluid up and cause the ink streams to cross each other; streaklines *can* cross each other as well

OUTCOME:

Identify and differentiate between streaklines, streamlines, and pathlines

TEST YOURSELF

Determine when these three types of lines are the same; when are they different? (HINT: what would need to be true about the underlying flow, for the firefly to "change its mind", or our ink streams to follow different trajectories from each other?)

It is interesting to visualize how/why these lines are different using an example.

EXAMPLE

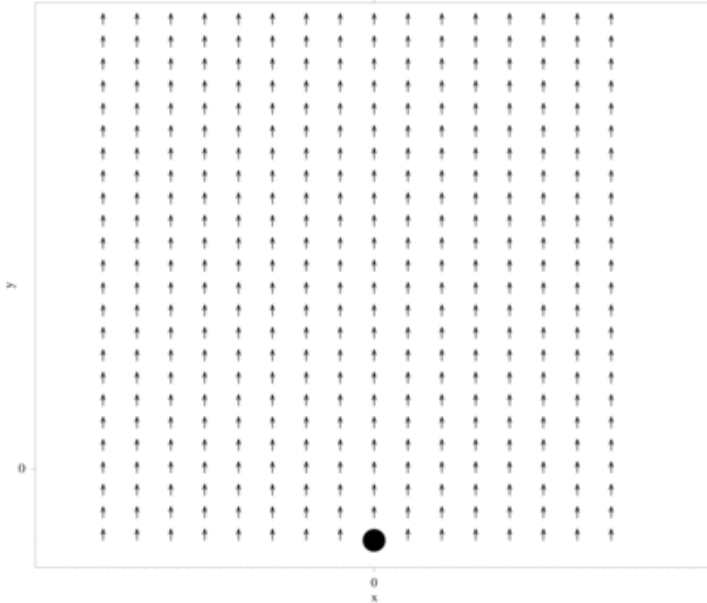
Given a time-dependent velocity field such as $v_x = \sin(t)$ and $v_y = 1$ it is instructive to look at how each of these visualization lines look/evolve:

ITP: Streamlines, Streaklines, and Pathlines Applet

It is interesting to visualize how/why these lines are different using an example.

EXAMPLE

Given a time-dependent velocity field such as $v_x = \sin(t)$ and $v_y = 1$ it is instructive to look at how each of these visualization lines look/evolve:



show pathlines show streamline show streaklines

There are a few interesting observations that can be made based on this exercise:

- All three lines are different
 - Only the pathline is steady (but, of course, it *has to be* as it shows the path that a single particle/fluid element actually takes)
 - Looking at the vector field and/or the streamlines give no good impression of what a pathline might actually look like. This is particularly important to note for **mixing applications** as many do not realize that mixing should really be thought of in a Lagrangian sense (like using a pathline) and that a purely Eulerian view (like a streamline) gives very little insight!
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ITP: Calculating Streamlines, Streaklines, and Pathlines

While streamlines, streaklines, and pathlines are primarily used as *visualizers*, clearly one needs to be able to calculate the actual lines in order to plot them.

Taking the following mathematical definitions, we can see how one could calculate these lines from knowing the underlying Eulerian description of the flow:

*A **pathline** is the easiest, being given as:*

$$\frac{d\vec{x}}{dt} = \vec{v}(\vec{x}, t)$$

This means that getting the x trajectory of the pathline is simply obtained by integrating the v_x expression. A **line** is generated from this data by eliminating the t variable from the x, y, (and z) trajectory expressions.

*A **streamline** is almost as easy, but we must recall that here we want the **instantaneous** flow field. Clearly, we cannot integrate with respect to t then (because we would then get trajectories moving into the future), instead we have:*

$$\frac{d\vec{x}}{ds} = \vec{v}(\vec{x}, t)$$

This means that integrating the v_x expression, but based on s! In other words, we treat t like a constant. This way we get x trajectories for a particular time as a function of s. Eliminating the s from our x, y, (and z) expressions again yields a **line**.

***Streaklines** are the hardest. recall here that we need to "connect the dots" for all points (x, y, z) that correspond to fluid elements that passed through a specific point (x₀, y₀, z₀) for times before now. So, we need to use the pathlines....*

- Rearrange the pathline trajectory expressions to solve for the "initial" positions (x₀, y₀, z₀) so that we have an expression for all of the positions x' of that particle at future times, t'.
- Plug this new expression for initial conditions in to replace the generic (x₀, y₀, z₀) that we had in the original pathline expressions
- Eliminate the t' in order to order to obtain the **lines** corresponding to streaklines

OUTCOME:

Mathematically derive streaklines, streamlines, and pathlines from an Eulerian velocity field.

TEST YOURSELF

Calculate the pathlines, streamlines, and streaklines for the applet example used earlier. That is, for a velocity field of $v_x = \sin(t)$ and $v_y = 1$. Let's start with the pathlines ...

***Pathlines** are calculated using:*

$$\frac{d\vec{x}}{dt} = \vec{v}(\vec{x}, t)$$

In our case this means that we need to integrate the v_x and v_y equations:

$$\frac{dx}{dt} = \sin(t) \text{ and } \frac{dy}{dt} = 1 \text{ give}$$

$$\int dx = \int \sin(t) dt \text{ so that } x = x_o + 1 - \cos(t) \text{ and}$$

$$\int dy = \int dt \text{ so that } y = y_o + t$$

we then eliminate t from the equations to give the curve:

PATHLINE

$$x = x_o + 1 - \cos(y - y_o)$$

Streamlines are calculated in much the same way except that we **fix** the time at some value, t , and then integrate against a "dummy" time, s :

$$\frac{d\vec{x}}{ds} = \vec{v}(\vec{x}, t)$$

In our case this means that we need to integrate the v_x and v_y equations:

$$\frac{dx}{ds} = \sin(t) \text{ and } \frac{dy}{ds} = 1 \text{ give}$$

$$\int dx = \int \sin(t) ds \text{ so that } x = x_o + \sin(t)s \text{ and}$$

$$\int dy = \int ds \text{ so that } y = y_o + s$$

we then eliminate s from the equations to give the curve:

STREAMLINE

$$x = x_o + (y - y_o)\sin(t)$$

For **streaklines** we start with our solution to the **pathline trajectories** (before we eliminated t):

$$x = x_o + 1 - \cos(t) \text{ and } y = y_o + t$$

We need to rearrange them so that they tell us what initial position (x_o, y_o) was occupied by some particle that is currently at position (x', y') at time t' :

$$x_o = x' - 1 + \cos(t') \text{ and } y_o = y' - t'$$

If we then use these "values" as our initial positions in the pathline trajectories, we can see where (else) that particle went (for all the other values of time t):

$$x = [x' - 1 + \cos(t')] + 1 - \cos(t) \text{ and } y = [y' - t'] + t$$

Last, we eliminate the t' so that we have streaklines for particles that have visited point (x', y') :

STREAKLINE

$$x = x' + \cos(y' - y + t) - \cos(t)$$

NOTE

In order to prove to ourselves that these three lines are the same for steady flows we need to note that (x', y') could easily be chosen to be (x_0, y_0) , since clearly if we had chosen $t' = 0$ that would fit our definition of (x', y') .

ITP: Eulerian and Lagrangian

Representing the properties of a fluid mathematically is a complex business as different "bits" of fluid occupy different points in space at different times (and the properties of interest *themselves* are likely to change in space or time or both)

There are two generally accepted descriptions of fluid properties as a function of position and time:

DEFINITION

*In an **Eulerian description** the fluid motion is described by calculating all of the necessary properties (e.g. density, pressure, velocity, etc) as functions of space (i.e. "fixed" positions in x, y, z space) and time. The flow itself is characterized by what happens at fixed points in space as fluid flows past these points.*

DEFINITION

*In a **Lagrangian description** the fluid motion is described by following individual fluid particles and noting how all relevant fluid properties change as a function of time as seen by the fluid particle being "followed".*

If "B" represents a particular fluid property (like temperature):

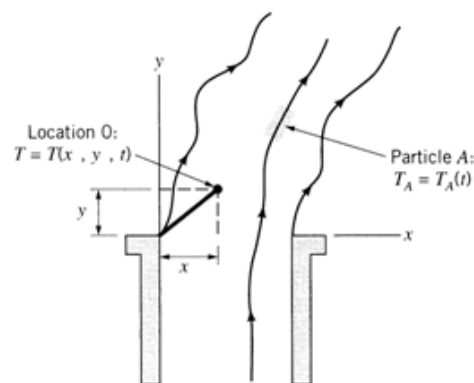
Eulerian description: $B = B(x, y, z, t)$

Lagrangian description: $B = B_A(t)$ or $B = B(x_{A0}, y_{A0}, z_{A0}, t)$

where fluid particle "A" can be identified by either its "name" (A) or its initial coordinates (x_{A0}, y_{A0}, z_{A0})

EXAMPLE:

Eulerian vs. Lagrangian description of temperature field:



OUTCOME:

Explain the difference between an Eulerian and Lagrangian viewpoints

TEST YOURSELF

If you throw a feather into the air and track its velocity will this give you a(n) Eulerian/Lagrangian description of the air's velocity? What if you were to measure the rotational speed of a bunch of windmills in a field?

If you assume that the feather is being moved as a passive tracer then you will obtain a Lagrangian description of the air velocity. That is, you will get

$$\vec{v}(t) = \vec{v}(x_0, y_0, t)$$

where $\vec{v}(t)$ is the time evolution of the velocity of the fluid that passed through point (x_0, y_0) (the initial location of your feather).

The windmills, on the other hand, will give you an Eulerian description including a full velocity field whereby the velocity at a bunch of locations (x, y) are known at all (observed) values of t , giving $\vec{v}(x, y, t)$.

ITP: Systems and Control Volumes

In order to conceptualize and perform balances, one must choose a representative "region" to perform the balance *on*. This "region" may be:

- fixed in space (Eulerian)
- moving in space (Eulerian)
- "attached" to a particular piece of mass (Lagrangian)

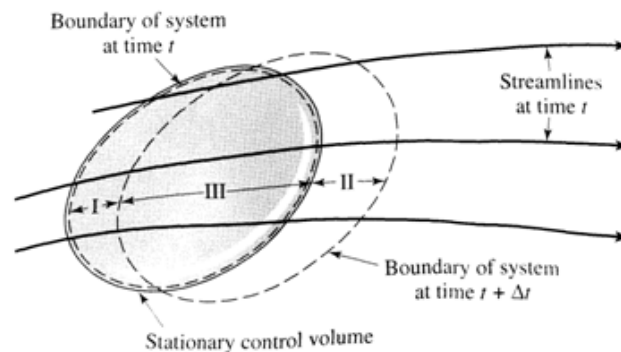
so that regions to "anchor" a balance can be grouped as either:

DEFINITION

A **System** is a collection of fluid matter of fixed identity (i.e. always the same fluid particle(s)) that will move, flow, and interact with its surroundings. (Lagrangian concept)

DEFINITION

A **Control Volume** is a geometrically defined volume in space through which fluid particles may flow (in or out of the control volume). Hence a CV in a flowing fluid will contain different fluid matter (fluid particles) at different points in time. (Eulerian concept)



OUTCOME:

Define systems and control volumes and identify when each is a useful frame of reference

TEST YOURSELF

Give an example of a balance that might be simpler using a CV? a System?

ITP: Dimensional Analysis

There are several very good reasons to regard engineering problems, not in terms of dimensional quantities, but instead as dimensionless equations/relations:

- make the equations/solutions "general"
- allow for the use of correlations
- allow comparisons/estimates of each terms "importance" (i.e., aid in the interpretation of the problem)

DEFINITION:

Dimensional analysis is the process by one predicts and identifies the number and form of all relevant dimensionless parameters for a given problem.

As stated above, dimensional analysis is a formal method of predicting *which* and *how many* dimensionless groups are important (and independent!) in a given problem.

The procedure for doing so is rather simple (and is called the Buckingham Pi method):

1. Make a list of relevant parameters:

Here you brainstorm on each of the material and process variables that may have some effect on your problem of interest.

2. Identify the *base units* represented in each of these variables:

In this step you take the variables from step 1, which usually have some form of compound units (like force) and break them into their base units/dimensions (like mass*length per time squared).

3. Use the Buckingham Pi *Theorem* to determine the number of independent dimensionless variables to make:

If you take the number of variables, V (from step 1), and subtract off the number of base dimensions, D , you get the number of independent dimensionless groups, G , possible:

$$G = V - D.$$

4. Chose a subset of your variables, V , equal to the number of dimensions, D , that are *recurring variables* for building your dimensionless groups:

This is the trickiest step for two main reasons: a) you need to choose these variables well (meaning that they represent all of the base dimensions in an independent way, if possible -- *avoid repeating the same type of variable!*) or else you may have difficulty getting reasonable groups; b) the variables you choose will actually affect your answer (but you can always show how new dimensionless groups can be made via combinations of *other* dimensionless groups, so it is no problem).

5. With each of the remaining non-recurring variables form "dimensionless groups" that have undetermined powers:

Here you form $V-D$ candidate groups by taking each of your recurring variables to a different unknown power (that is different for each candidate group!) and multiply it by one of your non-recurring groups (usually to the first power).

6. Solve for your unknown powers:

Step five will give you D algebraic equations for the powers in each of the G candidate dimensionless groups. You get these equations by simply recognizing that the sum of the powers for each base dimension must be equal to zero for a dimensionless group. Solve

the handful of simultaneous algebraic equations and you get your real dimensionless group. repeat this for each candidate and you are done!

OUTCOME:

Perform simple dimensional analysis, using the Buckingham Pi method.

ITP: Reynolds Number

Perhaps the most "famous" dimensionless group, but by no means the most "important", is the Reynolds number.

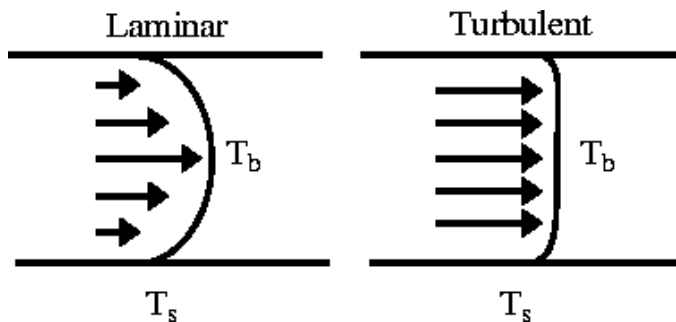
DEFINITION:

*The **Reynolds number** is a dimensionless group that relates the inertial forces to the viscous forces, and is written as:*

$$Re = \frac{\text{inertial force}}{\text{viscous force}} = \frac{UD\rho}{\mu} = \frac{UD}{\nu}$$

The most important function of the Reynolds number, not surprisingly, is to determine when inertial forces are bigger/smaller than viscous forces (which translates into whether we have a Stoke's flow (viscous-dominated flow) or not).

Perhaps the most employed function of the Reynolds number, however, is as an empirical measure of whether a flow is laminar or turbulent. Let's take a pipe flow as an example: In a pipe flow, the velocity profile varies depending on whether the flow is laminar or turbulent:



We get the flow on the left if $Re < 2300$ and the flow on the right if $Re > 5000$. (Where did these numbers come from?! What happens at $Re = 3400$?)

NOTE:

It is critical to note that these values of Re are empirical and refer only to flow in a pipe. Other transition values (for different geometries) are equally obscure and must be looked up (for example, flow past a sphere becomes turbulent at $Re = 340,000$!)

OUTCOME:

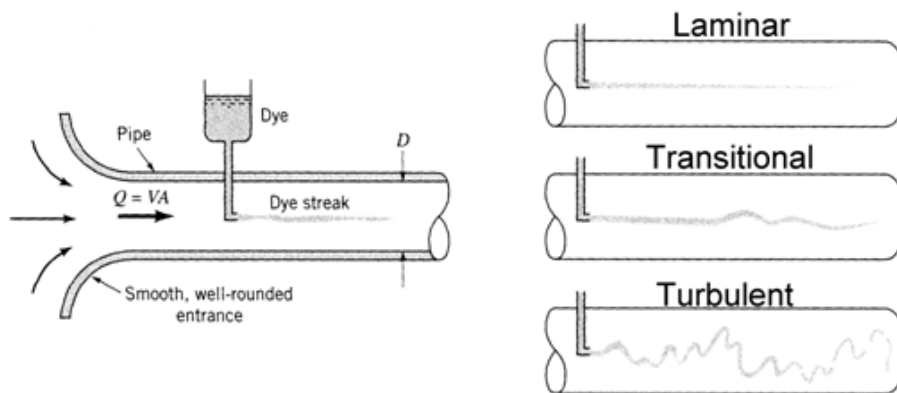
Calculate the Reynolds Number and use it to predict flow regimes

ITP: Laminar vs Turbulent

You may recall as a child walking in a nice straight line as a group of grade-schoolers. As you got older, you tended to travel a bit faster, and it was harder to keep the group orderly. Now, once class is finished, you all run pell-mell from the room in an almost random mob.

As we will discuss in the next lecture fluid flow involves transfer of momentum, whose resistance arises from friction within the fluid and with the surroundings (walls). If the resistance is sufficiently small, the fluid momentum is unbalanced and it "loses control" of itself, much like my students after class. In reality, both "mobs" stop being orderly because at high values of momentum any small fluctuation from orderly becomes magnified and difficult to bring back under control (since the resistance is too small to damp it out).

This leads us to the fact that there is generally thought to be three (really four) "regimes" of fluid flow:



DEFINITION:

Laminar flow is a "well-ordered" flow where adjacent fluid layers slide smoothly over one another (past one another) and interaction (material mixing) between these layers or lamina of fluid occurs only at the molecular level (i.e. as viscous stresses). Occurs at "lower" flowrates.

DEFINITION:

Turbulent flow is a flow characterized by random motion of fluid elements where each fluid element's velocity has a fluctuating nature. Occurs at "higher" flowrates and is often exploited for better mixing.

DEFINITION:

Transitional flow is a flow which exhibits both laminar and turbulent flow characteristics. Occurs at "intermediate" flowrates where the flow is transitioning from a purely laminar to purely turbulent regime.

Above, I mentioned that there is really four regimes of fluid flow because there is a flow that might be thought of as "super-laminar", often called Stoke's flow. This type of flow occurs when the damping force of "friction" (viscous effects) is so large that the inertia of the fluid is negligible. We will come back to this later.

OUTCOME:

Define laminar, turbulent, and transition flow regimes.

ITP: External versus Internal flows

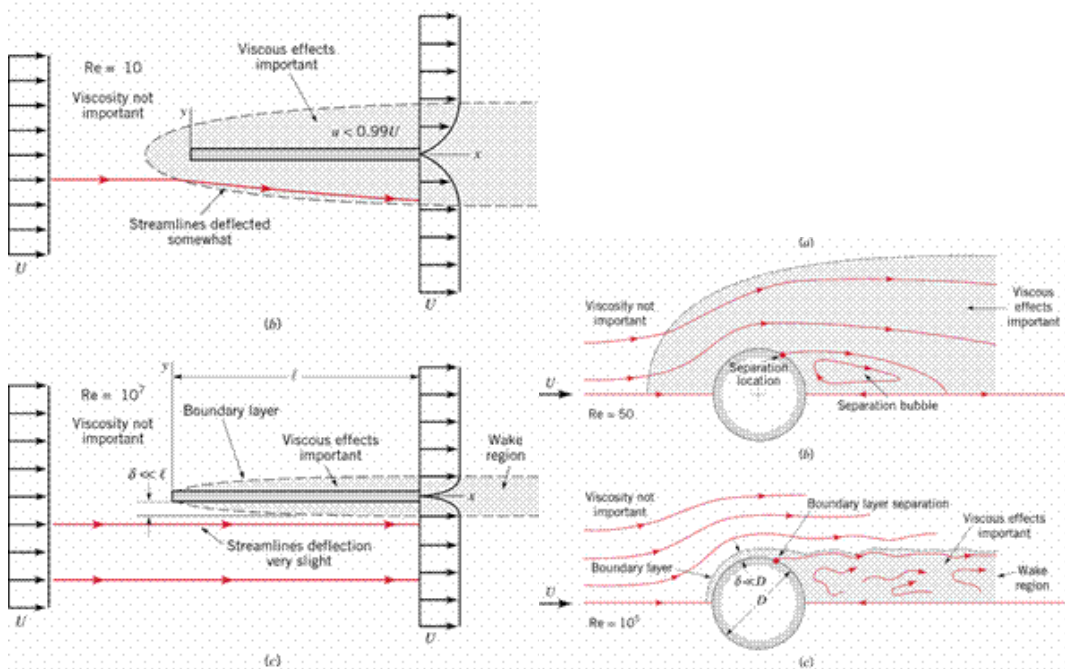
In all three types of transport phenomena it is quite important to distinguish between external and internal flows.

DEFINITION:

A **closed conduit** is any structure for conveying fluids from one point to another where the flow is not open to atmosphere along the path except perhaps at the end of the conduit. Examples include pipes, tubes, ducts, etc.

DEFINITION:

An **internal flow** is a process where fluid flows through the inside of a closed conduit. Typically, a pressure force (i.e. pressure gradient, pressure drop) or gravitational force is used to move the fluid and overcome the viscous shear forces (friction) offered by the walls of the conduit.



DEFINITION:

A **submersed object** is an object whose entire external surface area is in contact with a fluid flowing past the object.

DEFINITION:

An **external flow** is a process where fluid flows around a submersed object or a submersed object moves through a fluid (same problem, different reference frames)

The reason these types of flow are different is that the walls have a different effect in both cases. For external flows the wall introduces friction, mass, or heat into the flow, but that effect can keep traveling outward from the object indefinitely.

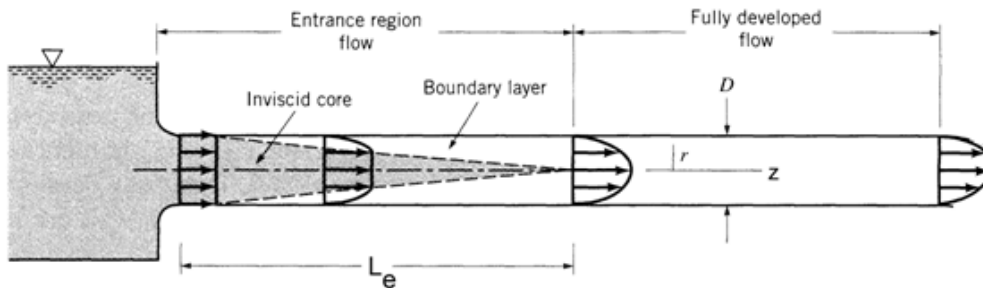
In an internal flow the walls begin to "gang up" on the fluid so that when they introduce friction, mass, or heat into the flow, these effects begin to "overlap" at some point.

OUTCOME:

Distinguish between external and internal flows.

ITP: Entrance Length

Now that we have defined external and internal flows, we need to point out that at the *beginning* of an internal flow we often have a "transition period" as the flow gets "used to" the idea of being an internal flow as opposed to an external flow.



DEFINITION:

The **entrance region/length** in a flow is the length of conduit necessary for the flow to transition from being an external flow to an internal flow. In the entrance region, the velocity changes in the direction of flow as it adjusts from the profile externally and at the inlet to a "fully-developed" profile where the effects of the walls are felt across the entire channel.

DEFINITION:

Fully-developed flow is what we call it when the velocity profile does not change along the direction of flow (axial direction), i.e. two different axial locations in conduit have same velocity profile.

As you might imagine the distance required as an entrance region depends on the width of the channel/pipe (as we need the frictional wall effects to "diffuse" to the center). What may not be obvious (yet) is that the necessary length *also* depends on the character of the flow (i.e., laminar vs turbulent), so that (for laminar flows in a cylindrical pipe):

$$\frac{L_e}{D} \cong 0.0575Re$$

for turbulent flows in a cylindrical pipe:

$$\frac{L_e}{D} \cong 4.4Re^{1/6}$$

NOTE:

These expressions will change slightly for different geometries. Also, if a pipe has a bend or other obstruction in it, we need to start all over again!

OUTCOME:

Explain the meaning of fully-developed flow and calculate the entrance region length necessary for a pipe flow.

Modes of Heat Transfer

As already mentioned, Thermodynamics alone cannot tell *how* or *how rapidly (rates)* heat transfer will take place.

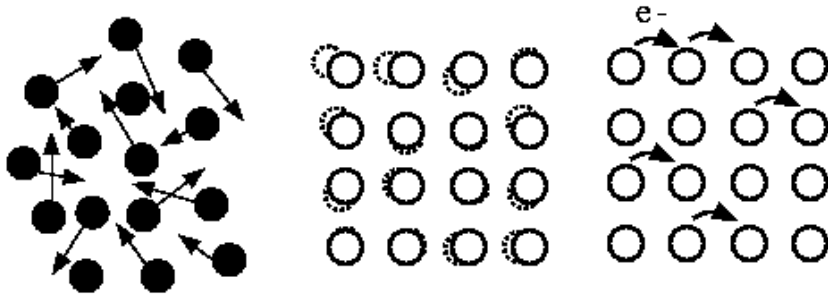
RATES -> how + driving force + resistance

Three "hows" (modes) of heat transfer: conduction, convection, and radiation.

Conduction

DEFINITION:

Conduction is the transfer of energy due to either random molecular motion or due to the motion of "free" electrons



Fluids
(liquids and gases)

Solids

Solids

In different phases of matter, the modes of conduction are slightly different:

- gases: conduction is due to collisions of randomly moving molecules.
- liquids: similar to gases, but with a *much* smaller "mean free path".
- solids: conduction is due to lattice vibrations and/or motion of "free" electrons.

Convection

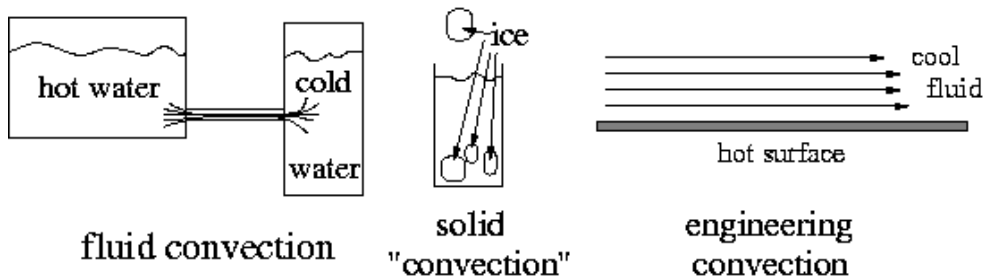
DEFINITION:

Convection refers to any transfer of thermal energy by motion of a medium.

In this sense, convection can refer equally well to a fluid (gas or liquid) flowing along or to a chunk of a solid being transported (perhaps thrown!).

NOTE:

In typical engineering application, convection is more broadly defined, so that it may also refer to transfer of thermal energy from a solid mass to a fluid flowing past that mass (where clearly conduction is also going on!).



A distinction is made between "forced" and "natural" convection.

Forced vs. Natural

- forced convection refers to the case when the fluid is made to flow by some external agent, like a pump for example.
- natural convection refers to fluid motion which naturally occurs from the heat transfer itself, due to buoyancy differences ("hot air rises...").

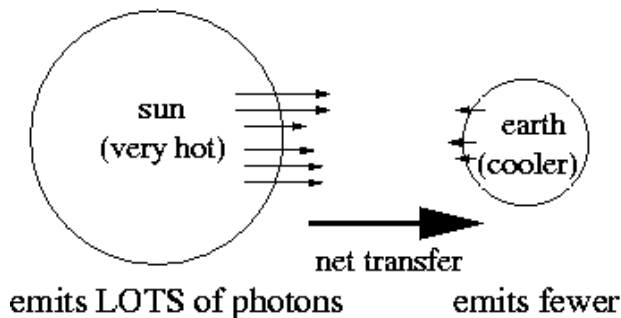
Radiation

DEFINITION:

In **radiative** heat transfer, objects emit and absorb electromagnetic waves/particles (photons).

NOTE:

There need not be any medium (mass) through which this form of heat is transported!



The amount of energy (photons) which is radiated depends on the temperature (thermal energy) of the radiator. (Radiator in this sense does *not* mean an apartment heater, it means a source of radiation. An apartment heater in fact acts more as a "convector" than a "radiator".)

Obviously, the way that heat is transferred in this mode is by an object emitting and absorbing *different amounts!* If a photon is absorbed the thermal energy of the mass *increases*, if a photon is emitted the thermal energy of the mass *decreases*. (Obviously, if something emits more than it absorbs it *cools down!*)

OUTCOME:

Explain and give examples of the three primary modes of heat transfer

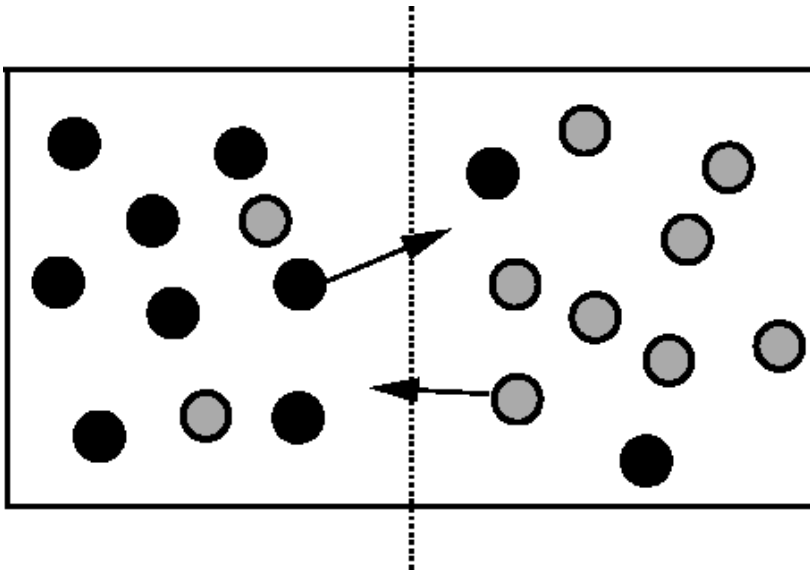
TEST YOURSELF

What modes of heat transfer dominate inside a oven? on a stove-top? in a "fry-heater" at McDonald's?

As in heat transfer there is a convective (flow-related) mode and a diffusive (conduction-like) mode.

Diffusion
DEFINITION:

Diffusion is the transfer of mass due to random molecular motion



The actual origin of diffusion is actually relatively easy to follow. Consider a box that (for some reason) has a high concentration of dark particles on the right and a high concentration of light particles on the left. If the particles are randomly jiggling around (which we know they will since they are at a temperature higher than 0 K), every now and then one will cross the center line (for simplicity, let's assume that two always cross at the same time, one going right and one going left, so that the overall concentration doesn't change).

If one of these "swapping" events occurs, it is quite likely (given the original concentrations) that it will move a light colored one from right to left and a dark colored one from left to right, causing a net flow of light particles from right to left and a net flow of dark ones from left to right. This same idea will hold true until both sides are at the same concentration. At which point swapping will not change the concentrations on either side (on average).

Convection
DEFINITION:

Convection refers to the transfer of mass due to an externally imposed flow.

NOTE:

This definition is slightly different than the convection in heat transfer because the flow here must be forced (there is no analog of "natural" convection for mass transfer -- although we will see something slightly like it later).

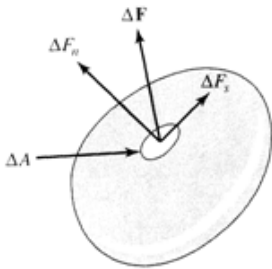
OUTCOME:

Describe the primary modes of mass transfer

ITP: Forces Acting on Fluids

Before we identify forces acting on fluids it is important to recall that we are interested (ultimately) in performing a momentum balance. Recall that momentum is mass X velocity; therefore if we are performing a differential balance on momentum, we will look at *rates* at which momentum flows in or out of our system/control volume (i.e., a momentum per unit time). Clearly momentum per unit time is equivalent to mass X acceleration, and therefore any differential momentum balance will necessarily balance forces on the fluid!

Stresses at a Point



Stresses in fluids can be of two types: normal and shear.

DEFINITION:

A **normal stress** is force per unit area acting normal (perpendicular) to a fluid area element ΔA , and is usually denoted as σ_{ij} .

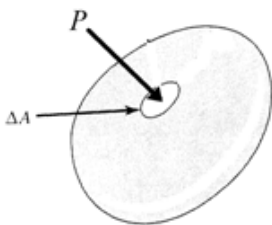
$$\sigma_{ij} = \frac{dF_n}{dA} \equiv \lim_{\Delta A \rightarrow \delta A} \frac{\Delta F_n}{\Delta A}$$

DEFINITION:

A **shear stress** is force per unit area acting tangential (parallel) to a fluid element ΔA , and is usually denoted as τ_{ij} .

$$\tau_{ij} = \frac{dF_s}{dA} \equiv \lim_{\Delta A \rightarrow \delta A} \frac{\Delta F_s}{\Delta A}$$

Fluid Pressure at a Point



The fluid pressure is one of the components of the normal stress. In fact, if the fluid is at rest, the pressure is *identically* the magnitude of the compressive normal stress.

It is important to note that the pressure always acts *compressively* on a fluid element so that while the *force magnitude* due to the pressure is given as:

$$dF_p = P dA$$

the *direction* of the force (i.e., the force vector) will always be in the direction opposite of the fluid element's normal vector (vector which points "outward" from a surface):

$$d\vec{F}_p = -P\vec{n}dA$$

NOTE:

Even in moving fluids the primary normal stress will be the pressure. Other normal stresses will be ignored except in Non-Newtonian fluids (we will define this term soon).

Forces Acting on Fluid in Control Volume

In an open system (i.e., one with flow into and out of the system), the forces acting on the fluid comprise three types:

- Pressure force at inlets and outlets (a surface force): $\vec{F}_p = -\int_{inlet}^{outlet} P\vec{n}dA$
- Pressure and shear force exerted by wall surfaces on fluid in CV (surface forces):

$$\vec{R} \equiv -\int_{walls} P\vec{n}dA + \int_{walls} \vec{\tau}dA$$

- Gravitational force exerted on all fluid within CV (a body or volumetric force):

$$\vec{W} \equiv \int_{CV} \rho\vec{g}dV = \rho V\vec{g}$$

OUTCOME:

Name and explain the origin of forces acting on fluids in a control volume.
