Linear Transport Relations (LTR)

Much of Transport Phenomena deals with the *exchange* of momentum, mass, or heat between two (or many) objects. Often, the most mathematically simple way to consider how and how fast exchanges take place is to look at *driving forces* and *resistances*.
In momentum transport, we are interested in driving forces that arise from differences in pressure and/or velocity.

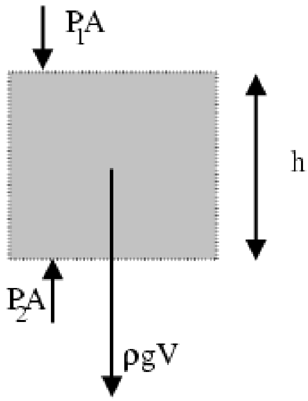
- Solve problems in fluid hydrostatics
  - Derive the pressure field equation [2.1]
  - Calculate the pressure distribution in a fluid or system of fluids that is at rest [2.2]
  - Use Archimedes' principle to calculate buoyant forces on (partially) immersed objects [2.3-2.4]
- Use friction factors and/or drag coefficients to calculate drag [12.2, 13.2-13.4]
  - Distinguish between lift, drag, skin friction, and form drag
  - Calculate friction factors from correlations and read friction factors off of charts
  - Use friction factors and/or drag coefficients to calculate drag on submersed objects (external flows)
  - Estimate friction losses in pipes and pipe networks

In heat and mass transport, our driving forces arise from differences in concentration and temperature.

- Perform convection and convection/radiation problems
  - Perform convective heat transfer calculations [15.3, 19.1, 19.2]
  - Perform convective mass transfer calculations [24.3, 28.1, 28.2]
  - Perform radiative heat transfer calculations [15.4, 23.1, 23.2, (23.7)]
  - Calculate the thermal resistance and magnitude of heat flow in combined convective/radiative systems [15.5]

LTR: The Pressure Field Equation

If a fluid element is at rest, we can do a simple force balance on the element (recognizing that the sum of forces equals zero for a non-accelerating "body"):



$0 = (P_1 - P_2 A) + \rho g V$

dividing through by area leaves:

$(P_2 - P_1) = \rho g h$

If we let the distance, h, get small so that the change in pressure ($P_1 - P_2$) also gets small, we get:

$dP = \rho g dz$

which we can rearrange to give:

$\frac{dP}{dz} = \rho g$

In three dimensions this gives us the Pressure Field Equation for a static fluid as:

$\nabla P = \rho \vec{g}$

NOTE:

Here gravity is a vector pointing in the z direction.

A more general form of this equation is possible if we relax the assumption that the fluid is not accelerating. In this case, we simply have the sum of the pressure and gravity forces equal to the mass of the fluid times acceleration:

$\rho V a = (P_1 - P_2)A + \rho g V$

again dividing through by area leaves:

$\rho h a = (P_1 - P_2) + \rho g h$

which for small h and P difference (in three dimensions) gives the general Pressure Field Equation:

$\nabla P = \rho(\vec{g} - \vec{a})$

OUTCOME:

Derive the pressure field equation

LTR: Calculating the Pressure Field

In a static fluid the pressure field equation is given as:

$\nabla P = \rho \vec{g}$

In order to solve this we need to recognize the meaning of the ∇ (gradient) operator.

DEFINITION:

*The **gradient** is a measure of the rate of change of a quantity in space (a slope). It yields a vector pointing in the direction of maximum spatial rate of change, so that the gradient of the temperature, for example, is (in cartesian coords):*

$$\nabla T = \frac{\partial T}{\partial x}\vec{e}_x + \frac{\partial T}{\partial y}\vec{e}_y + \frac{\partial T}{\partial z}\vec{e}_z$$

Using this definition, and taking z to be in the vertical direction, the component form of the Pressure Field Equation is written as:

$$\frac{\partial P}{\partial x}\vec{e}_x + \frac{\partial P}{\partial y}\vec{e}_y + \frac{\partial P}{\partial z}\vec{e}_z = \rho g \vec{e}_y$$

Rearrange and simplifying gives that the pressure is constant in the x and y directions, but varies in the z direction as:

$$\frac{dP}{dz} = \rho g$$

$$P_1 - P_2 = \rho g (z_2 - z_1) = \rho g h$$

NOTE:

While this looks exactly like where we started with our balance of forces, it is actually different because it is true for the whole fluid continuum, not simply for the small control volume on which we were doing the balance (this will make more sense when you do the Test Yourself, below).

OUTCOME:

Calculate the pressure distribution in a fluid or system of fluids that is at rest

TEST YOURSELF

Calculate the pressure distribution in a static gas. What is different in this case. As an example approximate the pressure distribution in the Earth's atmosphere.

$\nabla P = \rho \vec{g}$

Taking this as a 1D problem in rectangular coordinates (close anyway) and putting the origin at the surface of the earth with positive z upward, we get:

$$\frac{dP}{dz} = -\rho g$$

Assuming that air is an ideal gas, we can plug in $PV = nRT$ where we solve for $\rho = n / V$ so that $\rho = P / RT$:

$$\frac{dP}{dz} = -\frac{P}{RT} g$$

Rearranging and assuming that T does not change with z we get:

$$\frac{dP}{P} = -\frac{g}{RT}dz$$

which we can integrate from the surface of the earth (at $P = P_{atm}$) to some atmospheric height, H, where we will define $P = P$, to give:

$$\int_{P_{atm}}^{P} \frac{dP}{P} = -\frac{g}{RT}\int_{0}^{H} dz$$

$$P = P_{atm}e^{-\frac{gH}{RT}}$$

So far we have looked at the pressure and gravitational forces on a fluid element. What happens when we submerge a solid in the fluid?
Here we have the same pressure forces on top and bottom of a control volume of solid, as well as the gravitational force pulling downward on the solid; however, what is different is that we "subtracted" some fluid in order to put the solid in there.

DEFINITION:

Archimedes' Principle *states that the buoyant force, the force resulting from the "subtracted" fluid (whose volume is V_f), is equal to the weight of the displaced fluid in the direction opposite to gravity:*

$$\vec{F}_B = -\rho_f V_f \vec{g}$$

NOTE:

This force acts as a body force, going through the center of mass of the body, and therefore can be thought of as a density correction for the gravitational force (if the solid is completely submersed so that $V_f = V_s$):

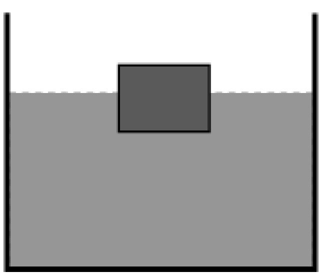
$$F_g = (\rho_s - \rho_f) g V_s$$

OUTCOME:

Derive the pressure field equation

TEST YOURSELF

Use Archimedes' principle to calculate buoyant forces on (partially) immersed objects



There are two forces acting on the block: the weight of the solid itself acting downward, the buoyant force (given as the weight of the displaced fluid). These forces must be equal if the block is not moving:

$$W = \rho_s g V_s = F_B = \rho_f g V_f$$

If we define f as the fraction of the block that is under fluid, then $V_f = f V_s$ so:

$$\rho_s g V_s = \rho_f g (f V_s)$$

which can be rearranged to give:

$$\rho_s = f \rho_f$$

Not surprisingly, as a fluid flows past a solid object it exerts a force on the solid. There are several classes of force typically discussed in fluid mechanics, but the two most common are drag and lift.

$$\vec{F}_T = -\int_A P\vec{n}dA + \int_A \vec{\tau}_w dA$$

DEFINITION:

*The **drag force** is the component of the force from the fluid on the solid that is in the direction parallel to the flow (here denoted as the x direction).*

$$D \equiv \vec{F}_T \bullet \vec{e}_x$$

DEFINITION:

*The **lift force** is the component of the force from the fluid on the solid that is in the direction perpendicular to the flow (here denoted as the y direction).*

$$L \equiv \vec{F}_T \bullet \vec{e}_y$$

In both the case of drag and lift the forces arise from two sources: fluid friction (viscous forces), and non-uniform pressure distributions (pressure forces).

NOTE:

Lift forces are only very weakly dependent on friction. they are primarily pressure-derived forces.
The components of the drag force that arise from friction and pressure are given "special names":

DEFINITION:

*The **skin drag/friction** is the portion of the drag force that arises due to shear stresses (viscous effects).*

DEFINITION:

*The **form drag** is the portion of the drag force that arises due to normal stresses (pressure effects).*

As the above equations would imply, the components of the drag may be analytically calculated if the pressure and shear stress distributions are known (and we will do this later); however, in many cases, these calculations are difficult and experimental measurements of these components are useful alternatives.

OUTCOME:

Distinguish between lift, drag, skin friction, and form drag.

In performing drag experiments, as with any transport experiments, it is useful to calculate/measure *dimensionless* quantities (for reasons of generality, scale invariance, etc.).

Taking the frictional force per unit surface area to represent the shear stress, we can make a dimensionless group using the fluid density and velocity to get:

DEFINITION:

*The **coefficient of skin friction** or the **Fanning friction factor** is the ratio of the total normalized (i.e., dimensionless) shear stress acting on the surface of a solid.*

$$f_f = C_f = \frac{F_s / A_S}{\frac{1}{2}\rho v_\infty^2}$$

NOTE:

The Fanning friction factor is the one most often used by chemical engineers

If instead we base our dimensionless group more on the "head losses" (a pressure-related drag to be discussed next) we get the **Darcy friction factor**, f_D. While the Darcy factor is more common in general engineering, it can simply be shown to be 4 times as big as the Fanning friction factor, so be careful which is which!

DEFINITION:

*The **drag coefficient** is a more general dimensionless group that is the ratio of the total normalized drag (i.e., not simply skin drag) acting on the surface of a solid. Here, we use the total projected area rather then the surface area.*

$$C_D = \frac{F_d / A_p}{\frac{1}{2}\rho v_\infty^2}$$

NOTE:

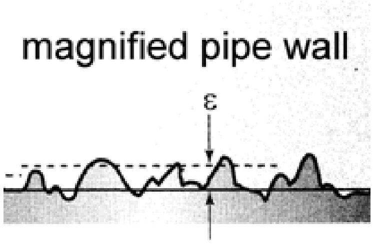
Typically in internal flows (pipe flows and friends) we are interested primarily in the skin drag, and use friction factors. In external flows, form drag plays a larger role and we tend to use the (total) drag coefficient.

Correlations and Charts

For laminar flow in a pipe, we can analytically show that the (Fanning) friction factor is given by:

$$f_f = \frac{16}{Re}$$

In turbulent flows, however, things are more difficult, as the friction factor depends strongly on the pipe roughness:



For a very *smooth pipes* a correlation is given as:

$$\frac{1}{\sqrt{f_f}} = 4\log_{10}\left[Re\sqrt{f_f}\right] - 0.40$$

For turbulent flow in a rough pipe:

$$\frac{1}{\sqrt{f_f}} = 4\log_{10}\frac{D}{e} + 2.28$$

An expression for transitional flow is given by:

$$\frac{1}{\sqrt{f_f}} = 4\log_{10}\frac{D}{e} + 2.28 - 4\log_{10}\left(4.67\frac{D/e}{Re\sqrt{f_f}} + 1\right)$$

Perhaps the easiest way to obtain friction factors is to use the popular **Moody charts** (on page 173 of your text).

OUTCOME:

Calculate friction factors from correlations and read friction factors off charts

TEST YOURSELF

Calculate the friction factor for a water flow though a smooth 1in ID pipe at (an average of) 10 m/s.

First we calculate the Reynolds number:

$$Re = \frac{V_{avg}D}{\nu} = \frac{(10m/s)(1in)(2.54x10^{-2}m/in)}{1x10^{-6}m^2/s} = 2.54x10^5$$

For flow through a circular pipe, this would be considered a turbulent flow. Using the Moody charts, we get $f = 0.0036$.

Alternatively, we can use the correlation for a turbulent flow in a very smooth pipe:

$$\frac{1}{\sqrt{f_f}} = 4\log_{10}\left[Re\sqrt{f_f}\right] - 0.40$$

Using "solver" (or similar) and/or trial-and-error, we get $f \approx 0.0038$ using this method.

LTR: Drag in External Flows

In addition to the correlations for internal (pipe) flows discussed previously, there are a number of expressions and charts for drag on submersed objects (external flows) in the text.

One of the most famous drag formulas is that of Stoke's drag, used for a viscously dominated flow:

$$F_d = F_P + F_\mu = 2\pi\mu R v_\infty + 4\pi\mu R v_\infty = 6\pi^2\mu R v_\infty$$

NOTE:

This expression is useful for viscous fluids, but also for small particles (recall the definition of the Re!). Can you show that the drag coefficient in Stoke's flow is:

$$C_D = \frac{24}{Re}$$

As the flow becomes "faster" the character of the wake behind the blunt body changes and thus the form drag increases On Page 141 a chart shows the behavior of C_D as a function of Re. Why do you think it levels off?!

OUTCOME:

Use friction factors and/or drag coefficients to calculate drag on submersed objects (external flows) and for internal flows

TEST YOURSELF

Calculate the drag on a Nolan Ryan fastball.

As with just about all drag-related problem, we must first calculate the Reynolds number:

$$Re = \frac{VD}{\nu} = \frac{(100mph)(0.073m)\left(\frac{1609m/mile}{3600s/hr}\right)}{1.4x10^{-5}m^2/s} = 2.33x10^5$$

For flow past a sphere, we can get the drag coefficient from the chart on page 141: $C_D \approx 0.4$.

If we now want to calculate the drag *force* from this we use the definition of the drag coefficient:

$$C_D = \frac{F_d / A_P}{½\,\rho V^2}$$

Rearranging to get get the drag force, F_d:

$$F_d = ½\,\rho V^2 C_D A_P$$

which gives us:

$$F_d = (0.5)(1.2kg/m^3)(45m/s)^2(0.4)\frac{\pi(0.073m)^2}{4} = 2.4N$$

You may have noticed that the normalizing "drag pressure" used thus far has been $\frac{1}{2}\rho v_\infty^2$, which is often called the *dynamic pressure*. Another common way to denote "pressure" losses due to drag is to quote them as *head losses*.

DEFINITION:

*The **head loss** is the energy lost per unit weight of the fluid.*

Head losses for straight lengths of pipe are directly related to the friction factor and constitute "major losses" in pipe flow:

$$h_L = 2f_f \frac{L}{D}\frac{v_\infty^2}{g}$$

Head losses for bends and other pipe components are related to an empirical (experimentally measured) factor called the loss coefficient/factor, K_i (see page 175 for values in pipe bends) and constitute "minor losses" in pipe flow:

$$h_L = 2f_f \frac{L_{eq}}{D}\frac{v_\infty^2}{g} = K_i \frac{v_\infty^2}{2g}$$

Another examples of loss coefficient/factor is for pipe constrictions, where:

$$K_i = 0.45\left(1 - \frac{A_2}{A_1}\right)$$

or pipe enlargements, where:

$$K_i = \left(\frac{A_2}{A_1} - 1\right)^2$$

NOTE:

The terms "major" and "minor" losses do not necessarily denote which is larger, simply which is most common!

In order to use head losses to calculate pressure drops, we use the following equation (which we will derive later in the course):

$$\rho g h_L = \frac{\rho\left(v_1^2 - v_2^2\right)}{2} + \left(P_1 - P_2\right)$$

OUTCOME:

Estimate friction losses in pipes and pipe networks

TEST YOURSELF

You are analyzing the flow between the hot water heater and the shower in a ranch-style home (so that elevation changes may be neglected). Calculate the mainline water pressure when the flowrate is 1L/s and the water is flowing through 10 meters of 2cm ID (smooth) pipe, going through 3 90^o elbows and you may consider the shower-head equivalent to a wide-open angle valve. (Note: the fluid velocity does not change along the region of interest.)

In order to calculate the mainline pressure, we need to figure out the necessary pressure-drop in the system described. In order to obtain that, we need to calculate the *head losses* for each of the parts of the pipe network.

Our calculations, therefore, need to include contributions from:

- Straight pipe ("major") losses
- "Minor" losses from the elbows
- "Minor" losses from the shower-head

Head losses for the straight length of pipe is obtained from the friction factor, so we must first calculate fluid velocity so that we can get the Re:

$$Q = V_{avg}A_{cs} = 1L/s = \pi(0.01m)^2 V_{avg}$$

This gives V_{avg} = 3.18m/s, so that:

$$Re = \frac{(0.02m)(3.18m/s)}{5x10^{-7}m^2/s} = 1.27x10^6$$

Since the flow is strongly turbulent, we will assume that $V_{avg} = V_\infty$. Also, we get f_f from the charts to be $f = 0.0027$, so that:

$$h_L = 2f_f \frac{L}{D}\frac{v_\infty^2}{g} = 2(0.0027)\left(\frac{10m}{0.02m}\right)\left(\frac{(3.18m/s)^2}{9.81m/s^2}\right) = 2.98m$$

Head losses for both the bends and the shower-head are obtained from the loss factor, using:

$$h_L = K_i \frac{v_\infty^2}{2g}$$

Looking up the values of K_i, we get $K_{90^o elbow} = 0.7$, while $K_{nozzle} = 3.8$, so that the head losses from these four things (3 elbows and a nozzle) are:

$$h_L = (3*0.7 + 3.8)\frac{(3.18m/s)^2}{2(9.81m/s^2)} = 3.04m$$

NOTE:

The "minor" losses are bigger than the "major"! Also, we could not group them in this way if v_∞ was different for each.

Then, the total head losses are calculated simply by adding these up, whereby we can get the pressure drop from:

$$\rho g \sum h_L = \frac{\rho(v_1^2 - v_2^2)}{2} + (P_1 - P_2)$$

which in our problem becomes:

$$\rho g \sum h_L = (P_1 - P_2)$$

so that:

$$(1000kg/m^3)(9.81m/s^2)(6m) = (P_1 - P_2) = 58.9kPa$$

This means that the pressure in the line is about 160kPa (since the pressure at the nozzle end must be atmospheric).

As we already discussed, convective heat transfer refers to the transport of heat due to a moving fluid (and in the engineering sense, applies to the case of a fluid "carrying" heat away from a solid boundary). In either sense of the term, it is clear that the rate of heat transfer will depend on the character of the fluid flow.
The rate expression for convection was suggested by Newton in 1701, commonly called Newton's "Law" of Cooling, and has the form:

EQUATION:

$$\frac{q}{A} = h \Delta T$$

where h is the convective heat transfer coefficient. It is sometimes also referred to as a "film coefficient" since in some strong flows the temperature changes are confined to a relatively thin film (see below).

"Law" is in quotes above because it is perhaps more correct to think of this expression as an empirical (data fitting) definition of h rather than a law. It will depend on:

- the geometry of the solid boundaries (this will typically be known)
- the nature of the fluid (conductivity, heat capacity, density, etc. : we can look these up)
- the nature of the flow (fluid mechanics!)

NOTE:

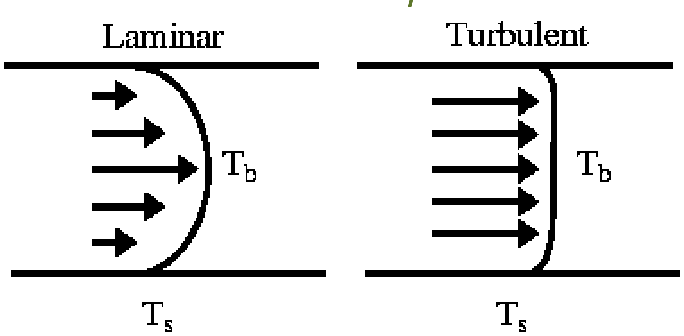
Determining the parameter, h, will often be the bulk of the work (or at least the only hard part) in a given convection problem.

OUTCOME:

Perform convective heat transfer calculations

EXAMPLE:

Lets look at an example:



Let us compare two cases of flow in a 1 m diameter pipe (a forced, internal, convection problem). In both cases the fluid is water ($\rho = 1000 kg/m^3$; $\nu = 1.25 x 10^{-6} m^2/s$). In both cases a suitable "bulk" temperature is 20C and the wall temperature is 40C. In the first case the fluid velocity is 1 mm/s, and in the second it is 1 cm/s. How does the flux of heat differ in these cases?

As in the conduction examples, we are simply interested in solving for the heat flow (not the temperature profile), so using Newton's Law may be sufficient (provided we know h). Assuming we know h, the governing equation is:

$$\frac{q}{A} = h(T_s - T_b)$$

Since we know the two temperatures, as mentioned before, the real problem is determining *h*. In this case, we can look up relations for flow in a pipe which tell us that: for fully developed laminar flow, there is an analytical solution:

$$h = 3.66\frac{k}{D}$$

where D is the pipe diameter. Here it is intuitive that the heat transfer coefficient goes up with higher fluid conductivity and goes down for a larger pipe. However, the convection coefficient **does not** depend on the velocity of the flow! (Which is weird to me anyhow. It is sufficient to say that the flow is laminar.)
for turbulent flow, there is an empirical relation:

$$h = 0.023\frac{V^{0.8}k^{0.6}(\rho c)^{0.4}}{D^{0.2}\nu^{0.4}}$$

where V is the fluid velocity. Again the heat transfer coefficient goes up with higher fluid conductivity and goes down (much less!) for a larger pipe, but now it *does* depend on the velocity of the flow as well as other properties of the fluid.
So, all we need to do now is determine the nature of the flow in our two cases. One important way to characterize fluid flows is to calculate the Reynolds number. The Reynolds number is a measure of the relative importance of the inertial and viscous forces in a flow.

$$Re = \frac{VD}{\nu}$$

In a pipe flow (*and only in a pipe flow*!), the flow is considered laminar for Re<2300 and turbulent for Re>5000.

Let us calculate Re's for our flows...

$$Re = \frac{(0.001m/s)(1m)}{1.25x10^{-6}m^2/s} = 800 \text{ (laminar)}$$

$$Re = \frac{(0.01m/s)(1m)}{1.25x10^{-6}m^2/s} = 8000 \text{ (turbulent)}$$

The heat transfer coefficients in each case are:

$$h = 3.66\frac{(0.56W/mK)}{(1m)} = 2.05W/m^2K$$

$$h = 0.023\frac{(0.01m/s)^{0.8}(0.56W/mK)^{0.6}((1000kg/m^3)(4200J/kgK))^{0.4}}{(1m)^{0.2}(1.25x10^{-6}m^2/s)^{0.4}} = 41.8W/m^2K$$

So,

$$\frac{q}{A} = (2.05 W/m^2K)(40^oC - 20^oC) = 41 W/m^2$$

$$\frac{q}{A} = (41.8 W/m^2K)(40^oC - 20^oC) = 836 W/m^2$$

What would happen if we did this same calculation for air rather than water?

NOTE:

*Often you will **not** be given any details of the fluid, the flow, (or even the geometry). Instead, you will simply be told what the "bulk" temperature in the fluid **is** as well as what the heat transfer coefficient (h) is. This is enough info to solve this type of convection problem.*

Convective mass transfer refers to the transport of mass due to a moving fluid. Like heat convection, this typically refers to transport *across phases*, however, here solid-fluid transport is on equal footing with liquid-gas transport (rather than being the dominant example of convection). As with heat transport, it is clear that the rate of mass transfer will depend on the character of the fluid flow.

In analogy with Newton's "Law" of Cooling, we can write an expression for the molar flux due to convection as:

EQUATION:

$$N_a = \frac{M_a}{A} = k_c \Delta C_a$$

where k_c is the convective mass transfer coefficient, N_a is the molar flux of species a, M_a is the molar flow of a, and A is the interphase area of contact.

NOTE:

We could write essentially the same expression based on mass concentrations, but will try to denote mass fluxes/flows with lower case letters. Also, for transport int he gas phase, we will often use partial pressures instead of molar concentrations.

As with heat transfer k_c may also sometimes be referred to as a "film coefficient".

k_c will depend on:

- the geometry of the phase boundaries (unlike heat transport, if we have gas-liquid transport this is a very difficult thing to calculate/measure!)
- the nature of the fluid (here the diffusivity)
- the nature of the flow (fluid mechanics!)

NOTE:

Again, determining the parameter, k_c, will often be the bulk of the work (or at least the only hard part) in a given convection problem.

OUTCOME:

Perform convective mass transfer calculations

EXAMPLE:

An aspirin sitting in your stomach has a solubility of 0.15 mol/L (so this is the concentration at the solid-liquid surface). Assuming that the concentration in the bulk of the stomach is zero and that the pill does not shrink, but stays a sphere with a 0.5cm diameter, calculate the molar flow into the stomach when the mass transfer coefficient is 0.1 m/s

As mentioned previously, radiation is unique in that no mass is necessary to bridge the gap between the two bodies which are exchanging heat. Instead, heat is transferred by emittance and absorption of energy "rays" or "packets" (photons). This is the way that the sun heats the Earth (clearly, as there is no mass between the two, in space).
The rate expression for radiation *emission* is associated with the names Stephan (1879) and Boltzmann (1884) who independently proposed the form:

EQUATION:

$$\frac{q}{A} = \sigma T^4$$

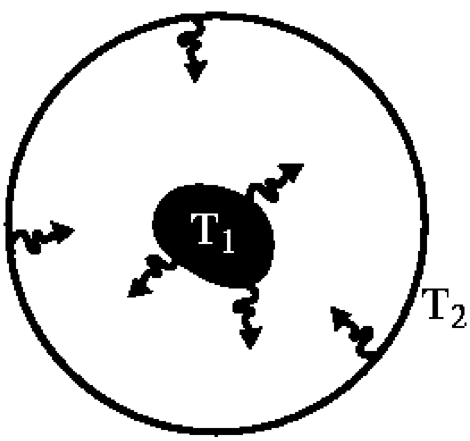
where σ is the Stephan-Boltzmann constant, $5.676x10^{-8} W/m^2K^4$. The above expression is only valid for the simplified case of what we will call a "perfect" radiator, called a "black-body" radiator.

NOTE:

Upon reflection (punny, eh ;), the choice of the name "black body" should not be surprising, as we have all experienced the fact that black shirts absorb solar energy more efficiently than white shirts. It turns out that they emit the energy more efficiently as well!

EXAMPLE:

Lets look at a very simple example:



Consider the situation where one black body (perfect radiator) is enclosed entirely within the other. We could imagine that body 1, at T_1, is radiating according to:

$$\frac{q}{A} = \sigma T_1^4$$

*Clearly **all** of this energy must hit body 2. Since body number 2, at T_2, is also radiating energy, we might assume for a minute that all of the energy radiated from body 2 not only hits body 1 but is absorbed by it so that:*

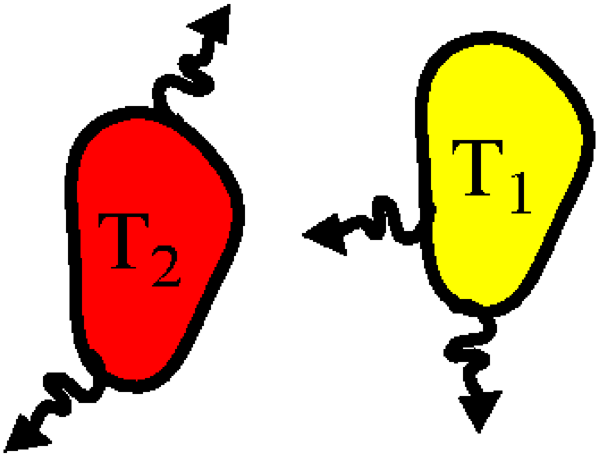
$$\frac{q}{A} = -\sigma T_2^4$$

This would lead us to a net heat flow from body 1 is given by:

$q_{1\rightarrow 2} = \sigma A_1(T_1^4 - T_2^4)$

We have actually oversimplified the real-life situation quite a bit in this example:

- we have used only black bodies
- we have assumed that all the energy which body 2 emits reaches body 1



If we instead consider the "realistic" bodies shown here, we can relax these two conditions.

Any Geometry

In order to relax our "aim" condition (that all energy emitted from 1 actually makes it to 2 and vice versa), we clearly need to consider the geometry of the problem (if some of body 1 faces *away* from body 2 the energy emitted from that side should *not* make it to 2 and vice versa).

These geometrical considerations are typically incorporated in a geometrical factor, F_{12}, called the view factor.

DEFINITION:

*The **view factor** is the fraction of the area of one body that is "seen" by the other body. Alternatively, we might think of it as the fraction of the radiation leaving one body that "hits" the other body.*

As you might gather from this definition, F_{12} is *direction specific*, meaning that it can only be used for heat transfer *from body 1 to body 2*. The product of F_{12} and A_1 is, however, *not* direction specific (i.e., $F_{12}A_1 = F_{21}A_2$). This fact is termed *reciprocity* and simply states that the net heat flow from one body to the other should not depend on which way it is calculated.

Going back to our first example, we might imagine that some of the radiation leaving body 2 actually hits body 2 rather than body 1. This means that F_{21} is *not* equal to 1; however, F_{12} =1 since all radiation leaving body 1 hits body 2, so our expression above is still correct (at least for black bodies in that geometrical configuration).

What is F_{21} in this case?

IMPORTANT:

*Two critical properties of the **view factor** then are:*

- $F_{12}A_1 = F_{21}A_2$
- $\sum_j F_{ij} = 1$

Non-Black (Gray) Bodies

Things become considerably more complex if we consider that not all bodies are perfect radiators. A simple method of relaxing the condition of non-black bodies is to introduce a correcting factor for the amount of energy emitted relative to that of a black body, ε (emissivity), so that

$$\varepsilon = \frac{E}{E_b}$$

where E is the emissive power of the material (at some temperature, T) and E_b is the emissive power of a black body (see the equation for the heat flow (the flux times the area) for a black body (above), and remember that power is energy per time....heat flow!). Using this relation, we can write that the heat emitted from body 1 is

$$\frac{q}{A} = \varepsilon\sigma T_1^4$$

Similarly, we can write a correcting factor for the absorption, α (absorptivity), so that the heat absorbed is

$$\frac{q}{A} = -\alpha\sigma T_2^4$$

It is often assumed that for a "gray" body, one in which ε and α are independent of temperature, the emissivity should be equal to the absorptivity ($\varepsilon = \alpha$). Even this simply rescaling of the radiation flux, however, makes radiation problems considerably more difficult as we now need to consider not only the initially incident radiation, but also reflected radiation (and we need to include the view factors for each rebounded ray!). For the purposes of this class, we will focus on situations where body 2 is much larger than body 1 (so that we can neglect radiation reflecting off of body 2 and we can consider body 2 essentially black) so we can write the new heat flux from body 1 to body 2 as

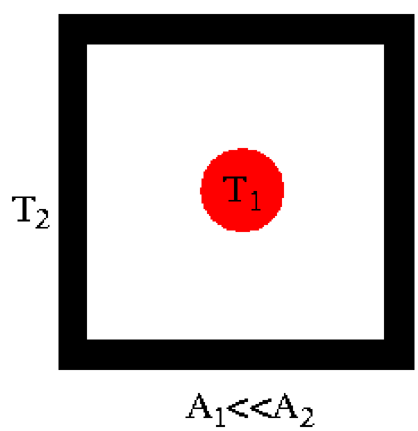
$$q_{1\rightarrow 2} = F_{12}\varepsilon_1\sigma A_1(T_1^4 - T_2^4)$$

OUTCOME:

Perform radiative heat transfer calculations

EXAMPLE:

A water cooled spherical object of diameter 10 mm and emissivity 0.9 is maintained at 80C when placed in a large vacuum oven whose walls are maintained at 400C. What is the heat transfer rate from the oven walls to the object? Recall that the



Stefan-Boltzmann constant,σ, is $5.676x10^{-8} W/m^2K^4$.

Since body 2 surrounds body 1 and $A_2 \gg A_1$, the governing equation (heat flow from 1 to 2):

$$q_{1\to 2} = \varepsilon_1 \sigma A_1 (T_1^4 - T_2^4)$$

where $A = \pi D^2$ (since it is the *surface area* of the sphere).
Plugging in our numbers

$$q = (0.9)(5.676x10^{-8} W/m^2K^4)\pi(0.01m)^2(353K^4 - 673K^4) = -3.04W$$

so for flow from walls (body 2) to sphere (body 1) $q = 3.04W$.
For simplicity, it is common to try to linearize the relation for radiative heat transfer (if you don't see why now, you will in a little bit!). So, if we factor the full equation, we get

$$\frac{q}{A} = \varepsilon\sigma(T_1 - T_2)(t_1 + T_2)(T_1^2 + T_2^2)$$

If we then assume that $T_1 \approx T_2$ (and use the mean, T_m) the sums can be simplified and the equation can be re-written

$$\frac{q}{A} \approx \varepsilon\sigma 4 t_m^3 (T_1 - T_2)$$

so that if we define h_r as

$$h_r = 4\varepsilon\sigma T_m^3$$

which leaves us with the final (very simple) form of

$$\frac{q}{A} = h_r(T_1 - T_2)$$

If we apply this equation to our previous problem we get for h_r

$$q = (27.6W/m^2K)\pi(0.01m)^2(353K - 673K) = -2.77W$$

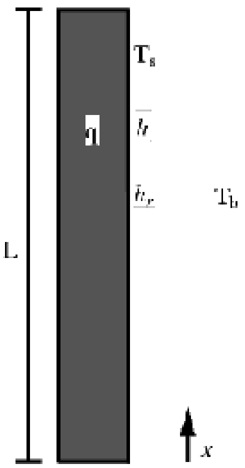
which is <10% off.

LTR: Combined Convection and Radiation

As is probably perfectly clear to you, it is of only limited value to understand these modes as purely separate entities. So we have explored how these things may act together.

EXAMPLE:

A room heater is in the form of a thin vertical panel of length, $L = 20cm$ and area, $A = 0.5m^2$. We will use an averaged temperature for the surface of the heater at 30C (303K). The air temperature far from the heater is constant at 20C (293K), and the emissivity of the heater is $\varepsilon = 0.9$. What is the rate of heat flow away from the heater?



We know that the heat leaving the surface of the "radiator" may leave as *either* radiation or convection.
So

EQUATION:

$$q = q_{rad} + q_{conv}$$

where we know that the two q's are given by:

$$q_{rad} = 4\varepsilon\sigma T_m^3 A \Delta T = h_r \Delta T$$

and

$$q_{conv} = hA \Delta T$$

Which gives us that:

$$q = hA \Delta T + h_r A \Delta T = (h + h_r) A \Delta T$$

Given that the convective heat transfer coefficent is $3.79W / m^2K$, we can then calculate our radiative heat transfer coefficient (with $T_m = 298K$):

$$h_r = 4\varepsilon\sigma T_m^3 = 4(0.9)(5.676x10^{-8}W / m^2K^4)(298K)^3 = 5.41W / m^2K$$
$$q = (3.79W / m^2K + 5.41W / m^2K)(0.5m^2)(303K - 293K) = 46W$$
