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## Analyzing Mass and Heat Transfer Equipment (MHE)

### **Analyzing Mass and Heat Transfer Equipment**

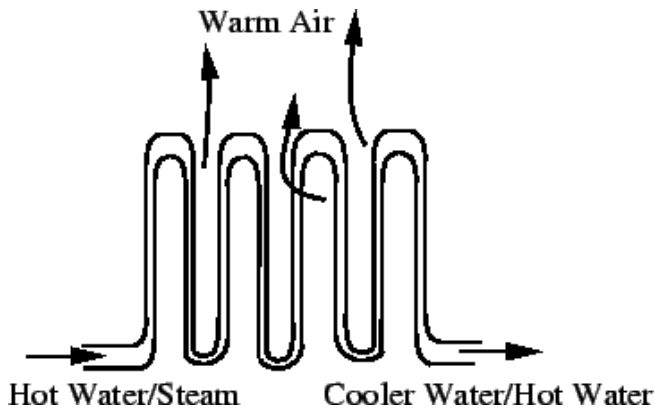
Scaling up to solving problems using process equipment requires both continuum and macroscopic knowledge of transport, and is industrially quite significant.

- Analyze heat transfer equipment
    - Distinguish between exchanger types [22.1]
    - Solve the "thermodynamic" problem [22.2]
    - Visualize the changing driving forces in co-current versus counter-current exchangers [22.2]
    - Calculate the "average" driving force from the heat flow and resistance [22.2]
    - Calculate the heat flow in an exchanger from the exchanger's effectiveness [22.4]
  - Analyze mass transfer equipment [31.5, 31.6]
    - Calculate the mass exchanged in a well-mixed contactor or the time necessary to achieve a particular exchange
    - Calculate the "average" driving force in a continuous contactor
    - Calculate the mass exchange in a continuous contactor and/or the size of the column
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## MHE: Heat Exchanger Types

While we have certainly seen numerous other applications of what we were learning as we went along, one of the most important applications of transport (for Chemical Engineers, at least) is the study/design/analysis of heat exchangers.

In general, a heat exchanger is a device that takes heat from one fluid stream and moves it to another. One example might be the "radiator" in your apartment. Hot water (steam) enters one side, by convection *to* the metal pipe, conduction *through* the pipe, and then convection *from* the pipe, the hot stream warms the exterior air and ultimately leaves as cooler water (or condensed, hot water).



As there is a very wide variety of heat exchanger designs, there is also a wide variety of ways to categorize them. For our purposes, we will be investigating heat exchangers (like the one above) where the two streams are not in direct contact (you should realize, however, that sometimes the streams *are* in direct contact in other types of devices). So, *our* heat exchangers typically have some solid surface between the two fluids. The fluids then can run in the same direction (co-current), in opposite directions (counter-current); they can pass each other once (single-pass), they can pass each other multiple times (multi-pass); both fluid streams can change temperature ("two-stream"), or only one of them might change temperature ("single-stream"). (This seems to be enough complication *without* considering the cases where the fluids actually are in direct contact!)

Some examples of these types of devices include: double-pipe, shell-and-tube, cross-flow, etc. Many of these can also be finned and/or baffled to increase surface area and/or fluid mixing.

### **OUTCOME:**

*Distinguish between exchanger types.*

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## MHE: Heat Exchanger Analysis (the "Thermo" Problem)



For a heat exchanger analysis, the first step is to do an overall macroscopic heat balance.

We are actually doing an energy balance that is simplified (we will see this soon). In other words, we assume that our exchanger is insulated, that none of our thermal energy is converted from or to other forms of energy, and that all of the heat that goes *in* must come *out* (just hopefully not in the same places!). We will tend to call this the *thermodynamic* part of the problem.

Using the average velocities (or assuming a plug flow) across all inlets and outlets and recalling that we (generally) have two of each, we can write this in a much simpler form as:

$$(\rho \hat{H} v A)_{H_{in}} - (\rho \hat{H} v A)_{H_{out}} = (\rho \hat{H} v A)_{C_{out}} - (\rho \hat{H} v A)_{C_{in}}$$

recalling that the mass flow rate is the product of density, average velocity, and cross sectional area, as well as the fact that  $\hat{H} = c_p T$ , we write this in its final form:

$$(\dot{m}cT)_{H_{in}} - (\dot{m}cT)_{H_{out}} = (\dot{m}cT)_{C_{out}} - (\dot{m}cT)_{C_{in}}$$

or

$$q = -(\dot{m}c \Delta T)_H = (\dot{m}c \Delta T)_C$$

### NOTE:

*This type of equation is probably familiar from your Thermodynamics class. Here, we have defined the **heat duty** as the total amount of heat that is transferred from one stream to the other. Also, we have used the convention that the change in temperature refers to  $T_{out} - T_{in}$  (hence the negative for the hot stream).*

### OUTCOME:

*Solve the "thermodynamic" problem*

### TEST YOURSELF

*A Light oil ( $c_p = 2100 \text{ J/kg K}$ ) is cooled by water in a heat exchanger. The oil flows at  $0.5 \text{ kg/s}$  and is cooled from  $375\text{K}$  to  $350\text{K}$ . If the water starts out at  $280\text{K}$  and flows at  $0.2 \text{ kg/s}$ . What is the water outlet temperature?*

Our macroscopic energy balance yields:

$$q = -(\dot{m}c \Delta T)_H = \dot{m}_H c_H (T_{H_{in}} - T_{H_{out}}) = (0.5 \text{ kg/s})(2100 \text{ J/kgK})(375\text{K} - 350\text{K}) = 26250 \text{ W}$$

Our macroscopic energy balance yields:

$$q = (\dot{m}c \Delta T)_C = \dot{m}_C c_C (T_{C_{out}} - T_{C_{in}}) = (0.2 \text{ kg/s})(4200 \text{ J/kgK})(T_{C_{out}} - 280\text{K}) = 26250 \text{ W}$$

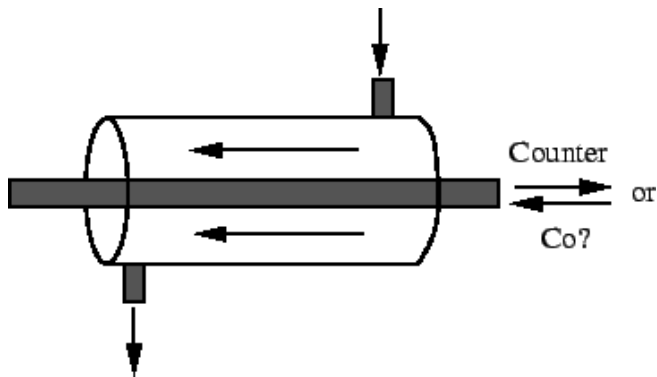
$$T_{C_{out}} = 311K$$

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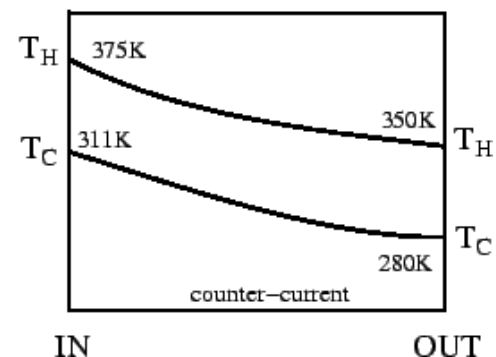
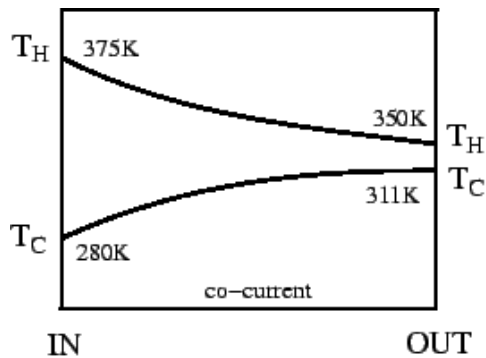
## MHE: Heat Exchanger Analysis (the driving force)



At this stage, we do not know whether things are running in counter or co-current flow. Being chemical engineers with a firm grasp of transport phenomena, however, we know that the *rate* of heat exchange is proportional to the driving force for the exchange (the temperature difference)! So, we might be interested in looking at how the two temperatures vary along the length of the exchanger.



In the previous example, we see that, while the two different cases yield different results, both seem feasible. (As we will see shortly, the only difference here would be the *area* need for the heat exchange).



However, if we modify our example problem, so that instead of cooling our fluid from 375K to 350K, we want to cool it to 310K. We see that the total amount of heat being transfer in the hot stream is:

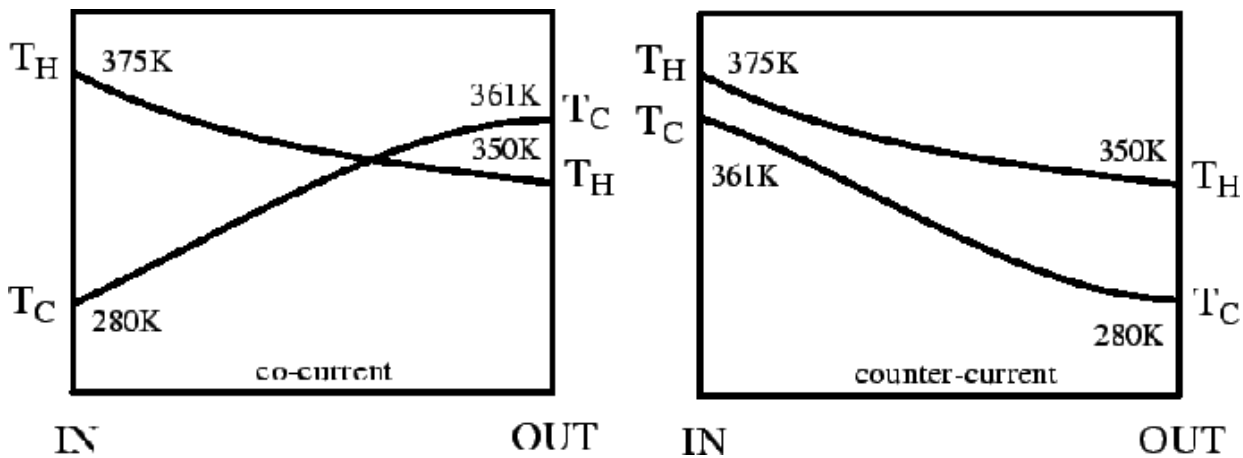
$$q = -(mc \Delta T)_H = \dot{m}_H c_H (T_{H_{in}} - T_{H_{out}}) = (0.5 \text{ kg/s})(2100 \text{ J/kgK})(375 \text{ K} - 310 \text{ K}) = 68250 \text{ W}$$

so that the water stream (which enters at 280K and 0.2 kg/s must also transfer that much, so its exit temperature is given as:

$$q = (mc \Delta T)_C = \dot{m}_C c_C (T_{C_{out}} - T_{C_{in}}) = (0.2 \text{ kg/s})(4200 \text{ J/kgK})(T_{C_{out}} - 280 \text{ K}) = 68250 \text{ W}$$

$$T_{C_{out}} = 361 \text{ K}$$

a result which makes perfect sense thermodynamically, but obviously limits our exchanger configuration, when we consider transport!

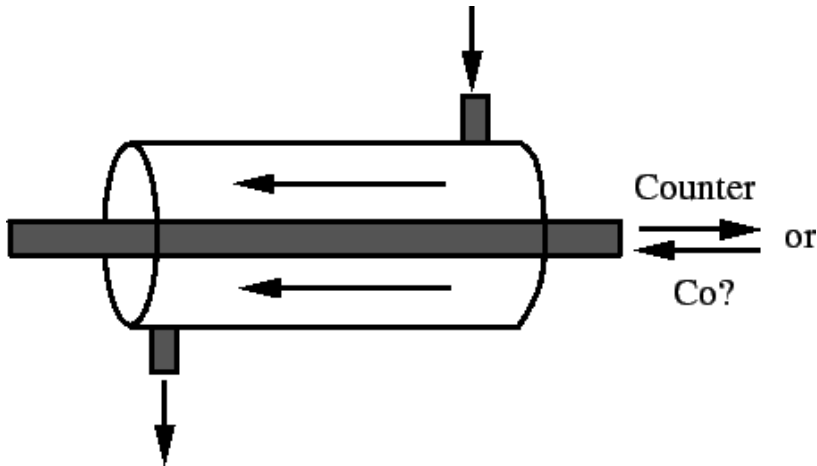


**OUTCOME:**

*Visualize the changing driving forces in co-current versus counter-current exchangers*

## MHE: The Log-Mean Temperature Difference

Now that we have looked at the *global* or *macroscopic* view of a heat exchanger, it is useful to look into more detail as to what goes on *inside* the black box...

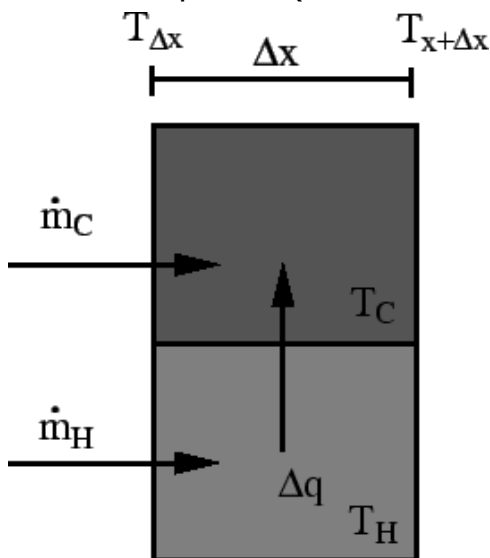


Before, we did a balance around the entire heat exchanger, if instead, we now look at a *microscopic* balance around a small portion of the exchanger (the following analysis will only be applicable (as it is) for a single-pass exchanger (either parallel or counter-current flow)):

If our small section has a length,  $\Delta x$ , and a perimeter  $P$ , we can say that the amount of heat transferred from the hot stream (at  $T_H$ ) to the cold stream (at  $T_C$ ), is given by:

$$q = UP \Delta x (T_H - T_C)$$

which is equal to (from a "macro" view of the section):



$$q = \dot{m}_H c_H (T_{H_{in}} - T_{H_{out}}) = \dot{m}_H c_H (T_{H_x} - T_{H_{x+\Delta x}})$$

so that

$$UP \Delta x (T_H - T_C) = \dot{m}_H c_H (T_{H_x} - T_{H_{x+\Delta x}})$$

dividing through by  $\Delta x$  and letting  $\Delta x \rightarrow \infty$

$$-UP(T_H - T_C) = \dot{m}_H c_H \frac{dT_H}{dx}$$

we also note that  $q = \dot{m}_C c_C (T_{C\_out}$ , so that we can write (by the same logic as above)

$$UP(T_H - T_C) = \dot{m}_C c_C \frac{dT_C}{dx}$$

dividing through by  $\dot{m}_C$  we get

$$-\frac{UP}{\dot{m}_H c_H} (T_H - T_C) = \frac{dT_H}{dx}$$

and

$$\frac{UP}{\dot{m}_C c_C} (T_H - T_C) = \frac{dT_C}{dx}$$

if we subtract the "hot-stream" equation from the "cold-stream" equation, we get

$$\frac{UP}{\dot{m}_C c_C} (T_H - T_C) + \frac{UP}{\dot{m}_H c_H} (T_H - T_C) = \frac{dT_C}{dx} - \frac{dT_H}{dx}$$

which we can rearrange to be

$$UP(T_H - T_C) \left( \frac{1}{\dot{m}_C c_C} + \frac{1}{\dot{m}_H c_H} \right) = \frac{d(T_C - T_H)}{dx}$$

dividing through by  $(T_H - T_C)$ , moving the  $dx$  to the other side, and multiplying both sides by a -1

$$\frac{d(T_H - T_C)}{(T_H - T_C)} = -UP \left( \frac{1}{\dot{m}_C c_C} + \frac{1}{\dot{m}_H c_H} \right) dx$$

if we integrate (Note: here, we are assuming the  $U \neq U(x)$ , but only a minor modification is necessary if  $U = U(x)$ ) from 0 to  $L$  (and denote the temperatures at those points as  $T_0$  and  $T_L$ ), we get

$$\ln \left( \frac{(T_{H,L} - T_{C,L})}{(T_{H,0} - T_{C,0})} \right) = -UPL \left( \frac{1}{\dot{m}_C c_C} + \frac{1}{\dot{m}_H c_H} \right)$$

going way back to the beginning, you should recall that  $q = C_H(T_{H,in} - T_{H,out}) = C_C(T_{C,out} - T_{C,in})$ , so plugging these two relations in for  $C_H$  and  $C_C$ , we get

$$\ln \left( \frac{(T_{H,L} - T_{C,L})}{(T_{H,0} - T_{C,0})} \right) = -UPL \left( \frac{T_{C,L} - T_{C,0}}{q} - \frac{T_{H,L} - T_{H,0}}{q} \right)$$

solving for  $q$  gives

$$q = -UPL \left( \frac{(T_{C,L} - T_{C,0}) - (T_{H,L} - T_{H,0})}{\ln \left( \frac{(T_{H,L} - T_{C,L})}{(T_{H,0} - T_{C,0})} \right)} \right)$$

which we can rearrange one last time to be

$$q = UPL \left( \frac{(T_{H,L} - T_{C,L}) - (T_{H,0} - T_{C,0})}{\ln \left( \frac{(T_{H,L} - T_{C,L})}{(T_{H,0} - T_{C,0})} \right)} \right)$$

if we define a log mean temperature,  $\Delta T_{LM}$ , as

$$\Delta T_{LM} = \frac{(T_{H,L} - T_{C,L}) - (T_{H,0} - T_{C,0})}{\ln \left( \frac{(T_{H,L} - T_{C,L})}{(T_{H,0} - T_{C,0})} \right)}$$

we can write our final result as

$$q = UPL \Delta T_{LM}$$

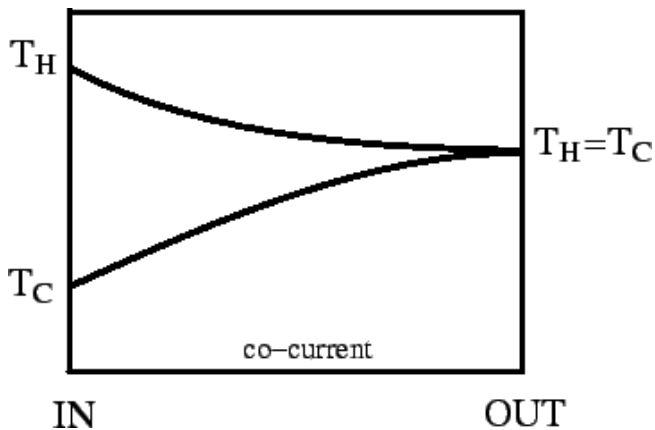
### OUTCOME:

Calculate the "average" driving force from the heat flow and resistance



## MHE: Heat Exchanger Effectiveness

One way of measuring a heat exchanger's performance is to calculate its effectiveness. The heat exchanger effectiveness,  $\varepsilon$  (note that this is *not* the same notation as the book), is defined as the ratio of the actual heat transfer to the heat transfer attainable in an infinitely long counterflow exchanger. We choose an infinitely long exchanger since that will yield the *maximum* heat transfer that can take place (i.e., eventually, enough heat will be transferred so that the driving force will *disappear* - the streams will reach the same temperature). To understand the rationale behind the choice of a *counterflow* exchanger (in this definition), it is instructive to look at both cases: an infinitely long parallel exchanger and one an infinitely long counterflow exchanger.

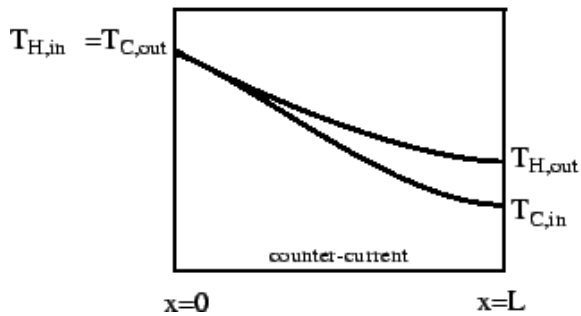
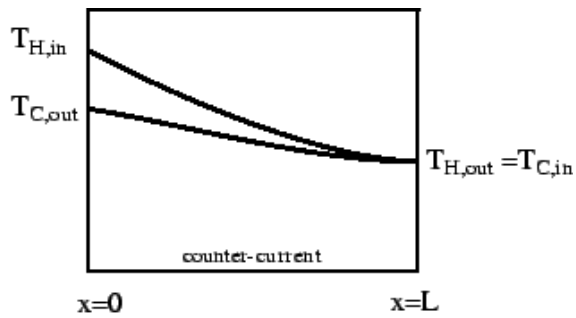


In the parallel flow case, the two fluids enter the exchanger with (typically known) inlet temperatures,  $T_{H_{in}}$  and  $T_{C_{in}}$  and leave the exchanger at the same (unknown) temperature  $T_M$  - which is somewhere in between  $T_{H_{in}}$  and  $T_{C_{in}}$ .

Now, in order to figure out the heat transfer (so that we might plug it into the denominator of our effectiveness definition) one needs to determine  $T_M$ . This can be calculated by equating the two heat duties,  $q = \dot{m}_C c_C (T_M - T_{C_{in}}) = \dot{m}_H c_H (T_{H_{in}} - T_M)$ , to give  $T_M = \frac{\dot{m}_H c_H T_{H_{in}} + \dot{m}_C c_C T_{C_{in}}}{\dot{m}_C c_C + \dot{m}_H c_H}$  (i.e., the weighted average of the temperatures). Plugging this solution into either of the above equations then tells us our heat duty.

$$q = \dot{m}_C c_C \left( \frac{\dot{m}_H c_H T_{H_{in}} + \dot{m}_C c_C T_{C_{in}}}{\dot{m}_C c_C + \dot{m}_H c_H} - T_{C_{in}} \right) = \dot{m}_H c_H \left( T_{H_{in}} - \frac{\dot{m}_H c_H T_{H_{in}} + \dot{m}_C c_C T_{C_{in}}}{\dot{m}_C c_C + \dot{m}_H c_H} \right)$$

An alternative is to operate the exchanger in counterflow. In this case, again, we will assume that we know the inlet temperatures,  $T_{H_{in}}$  and  $T_{C_{in}}$ . Also, as before there will (eventually) be no driving force (in an infinitely long exchanger). There are two ways in which this can happen: the hot stream can leave at the same temperature as the cold stream inlet, or the cold stream can leave at the same temperature as the hot stream inlet (clearly, if *both* of these happened at the same time the streams would have to be the same temperature *everywhere*! (this happens when the fluids have the *exact same* mass flow rate times heat capacity!)).



Which of the two cases actually happens in a given scenario is determined by which of the capacity coefficients is larger,  $\dot{m}_C c_C$  or  $\dot{m}_H c_H$ . If  $\dot{m}_C c_C > \dot{m}_H c_H$  then the hot stream will leave at the same temperature as the cold stream (and vice versa). It is easier to understand why if you consider the case where the fluid properties of the hot and cold streams are the *same* (except for temperature, of course), but the *flow rate* is vastly different (so that the capacity coefficients differ). Consider cold water rushing exceedingly quickly through the exchanger. It will change in temperature only a small amount since it will only be in the exchanger for a short time. In contrast if hot water is barely trickling through the exchanger it will have a *long* time to cool down and will eventually reach the temperature of the cold stream (in this example  $\dot{m}_C \gg \dot{m}_H c_H$ ). In any event, in order to determine the heat transfer (so that we might plug it into the denominator of our effectiveness definition) all we then need to know is whether  $\dot{m}_C c_C$  or  $\dot{m}_H c_H$  is larger since  $q = (\dot{m}c)_{\min} (T_{H,in} - T_{C,in})$  in either case (where  $(\dot{m}c)_{\min}$  is the *smaller* one).

So, by defining the effectiveness in this way, we have a much simpler expression for our effectiveness:

$$\varepsilon = \frac{\dot{m}_C c_C (T_{C,out} - T_{C,in})}{(\dot{m}c)_{\min} (T_{H,in} - T_{C,in})} = \frac{\dot{m}_H c_H (T_{H,in} - T_{H,out})}{(\dot{m}c)_{\min} (T_{H,in} - T_{C,in})}$$

### NOTE:

*In one of these two equations the  $\dot{m}c$  will cancel since either the cold or the hot must be the minimum fluid!*

*If, for any given exchanger, the effectiveness is known we can then write our heat duty equation as:*

$$q = \varepsilon (\dot{m}c)_{\min} (T_{H,in} - T_{C,in})$$

*without knowing the exit temperatures beforehand!*

### OUTCOME:

*Calculate the heat flow in an exchanger from the exchanger's effectiveness*



## HME: Types of Mass Transfer Equipment

Mass transfer equipment is necessary when you want to move one type of material from one phase to another. Examples of this type of operation are:

- Moving a solute from a gas into a liquid: adsorption, dehumidification, distillation
- Moving a solute from a liquid into a gas: desorption/stripping, humidification
- Moving a solute from one liquid phase into another: liquid-liquid extraction
- Moving a solute from a solid into fluid phase: drying, leaching
- Moving a solute from a fluid phase into a solid phase: adsorption, ion exchange

In order for efficient mass transfer to occur, it is necessary that one achieve intimate contact between the phases. For many of these operations towers are used: bubble towers, spray towers, packed towers. In other applications, a simple well-stirred vessel is used (like a CSTR: continuously stirred tank reactor).

### **OUTCOME:**

*Discuss the need for and describe the types of mass transfer equipment*

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## MHE: Analyzing Mass Transfer Equipment (tower-type, not mixed)

In order to analyze mass transfer equipment, we need to recognize that there are two ways that we may denote the amount of material being transferred:

- Simple mass balance(s)
- Transport rate equation(s)

Furthermore, we must recognize that the transport rate equations that we have encountered thus far for interphase transport, such as:

$$N_A = k_G(p_{A,G} - p_{A,i}) = K_G(p_{A,G} - p_A^*)$$

yield an answer with units of mass transferred per unit time per unit *perpendicular* area. In other words, this expression gives us an answer based on the area of contact between the two phases (a quantity that in many cases is immeasurable)

If we multiply these equations by  $a$ , the interfacial area per unit volume, and  $dz$ , a differential height section of a tower, we get an expression:

$$N_A a dz = k_G a (p_{A,G} - p_{A,i}) dz = K_G a (p_{A,G} - p_A^*) dz$$

which now has units of mass transferred per unit time per unit *cross-sectional* area. (Note: This step is essentially multiplying by the interfacial area,  $A_i = aV$ , and then dividing by the cross-sectional area,  $A_{CS}$ .)

### DEFINITION:

*The **capacity coefficient** is defined as the product of the convective mass transfer coefficient (either single-phase or interphase/overall) and the interfacial area per unit volume.*

Writing a mass balance for species A over a differential segment of a tower (denoting the two phase flows as L and G), and then dividing by cross-sectional area, we get:

$$\frac{G}{A_{CS}}(y_{A,z+dz} - y_{A,z}) = \frac{L}{A_{CS}}(x_{A,z+dz} - x_{A,z})$$

### NOTE:

*Here we have implicitly assumed that we have a dilute solution so that the total flows L and G do not change as species A is transferred.*

For a sufficiently small differential section, we can write this as:

$$\frac{G}{A_{CS}} dy_A = \frac{L}{A_{CS}} dx_A$$

Noting that either of these could be written as being equal to our modified flux (above; i.e., per unit cross-sectional area), we can write:

$$N_A a dz = K_G a (p_{A,G} - p_A^*) dz = \frac{G}{A_{CS}} dy$$

Using our dilute assumption again, we can write the partial pressures in such a way as to get:

$$N_A a dz = K_G a P_{tot} (y_{A,G} - y_A^*) dz = \frac{G}{A_{CS}} dy$$

Re-arranging and preparing to integrate gives us:

$$\int_0^L dz = \frac{G}{A_{CS} K_G a P_{tot}} \int_0^L \frac{dy}{(y_{A,G} - y_A^*)}$$

The problem with this expression is that  $y_A^*$  varies along the length (as the  $x_{A,G}$  varies). Because of this, the easiest way to handle this type of problem is to note the following:

For a dilute solution, we have already seen that the mass balance leads to a straight line (above), but the equilibrium expression will also lead to a straight line:  $y_A^* P_{TOT} = Hx_{A,G}$ , with a constant value of H.

With this linear dependence of both of the equations, we can re-write the above equation in terms of the *difference* between  $y_{A,G}$  and  $y_A^*$ , so that with:

$$\Delta = (y_{A,G} - y_A^*)$$

In order to make out change in variables, we can show that:

$$\frac{d\Delta}{dy_a} = \frac{\Delta_L - \Delta_0}{y_{A,L} - y_{A,0}}$$

therefore, our integral becomes:

$$\int_0^L dz = \frac{G}{A_{cs} K_G a P_{tot}} \frac{y_{A,G,L} - y_{A,G,0}}{\Delta_L - \Delta_0} \int_0^L \frac{d\Delta}{\Delta}$$

So that integrating gives:

$$L = \frac{G}{A_{cs} K_G a P_{tot}} \frac{y_{A,G,L} - y_{A,G,0}}{\Delta_L - \Delta_0} \ln \frac{\Delta_L}{\Delta_0}$$

and rearranging gives:

$$M_A = N_A a A_{CS} L = G (y_{A,G,L} - y_{A,G,0})$$

$$= K_G a P_{tot} L A_{CS} (y_{A,G} - y_A^*)_{LM} = K_G a V P_{tot} (y_{A,G} - y_A^*)_{LM}$$

**OUTCOME:**

Calculate the "average" driving force in a continuous contactor

**OUTCOME:**

*Calculate the mass exchange in a continuous contactor and/or the size of the column*

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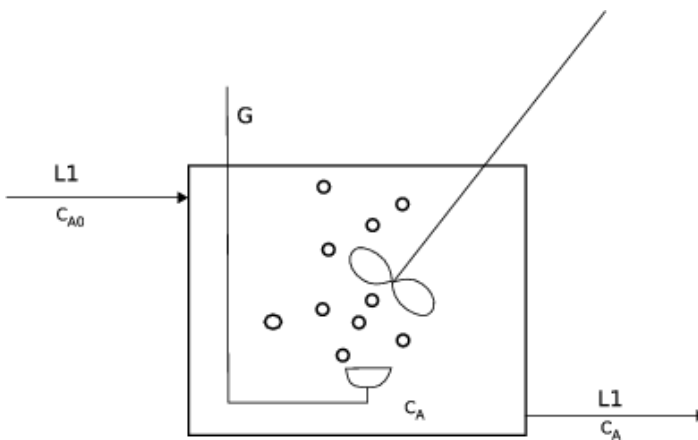
## MHE: Analyzing Mass Transfer Equipment (well-stirred)

In a well stirred contacting vessel, the mass exchange analysis is rather simple. For mathematical simplicity we will restrict our attention to dilute systems where we can ignore changes in *overall* material flows, as the concentrations change (i.e., the concentration changes are small enough that the resultant changes in total flow are negligible). This restriction is easily removed, but is an unnecessary complication.

### NOTE:

*In a well-mixed tank (sometimes called a CSTR: continuously stirred tank reactor), the concentration inside the tank is assumed to be uniform and equal to the concentration in the **exit** stream.*

### A Continuous Contactor



Performing a balance on species A in the liquid phase of the continuous process shown above, one can start with a "word equation":

rate of change of mass of species A = net rate of inflow of species A + net rate of reaction of species A

Assuming no reaction and steady state conditions, this reduces to

0 = net rate of inflow of species A

The inflow of A to liquid 1, can occur from two sources: (1) the actual inflow of the liquid phase and (2) transfer of species A between phases (**NOTE:** in both cases here we are talking about a *net* inflow, so each item could easily be negative and therefore technically be an outflow)

$$0 = L_1 C_{A_0} - L_1 C_A + N_A A_i$$

### NOTE:

*Here, the area,  $A_i$ , is the interfacial area which is the area perpendicular to the mass flux. This quantity is difficult (impossible) to measure so we instead use the product  $A_i = aV$ , where  $a$  is the interfacial area per unit volume, a quantity that can be experimentally determined and used directly for scale-up.*

Using  $aV$  for the interfacial area and plugging in for the expression for the mass flux,  $N_A$ , we get:

$$0 = L_1 C_{A_0} - L_1 C_A + K_L a V (C_A^* - C_A)$$

Recall that we use  $C_A$  in our expression for mass flux since that is the concentration everywhere within the contacting apparatus.

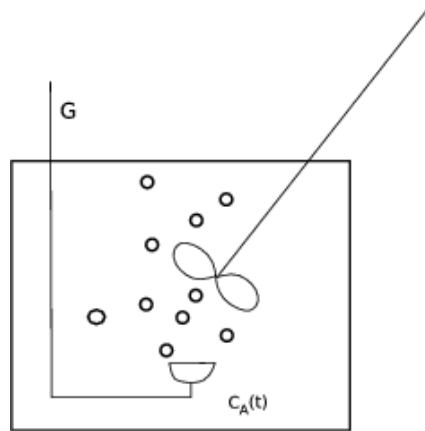
**DEFINITION:**

*The product,  $K_L a$ , is called the **capacity coefficient** and is often used as an experimentally determined parameter for scale-up of mass transfer equipment.*

Rearranging this expression, we get an equation for the outlet concentration for this type of contactor:

$$C_A = \frac{L_1 C_{A_0} + K_L a V C_A^*}{L_1 + K_L a V}$$

**A (semi-)Batch Contactor**



Performing a balance on species A in the liquid phase of the semi-batch process shown above, one can start with a slightly different "word equation":

$$\begin{matrix} \text{rate of change of} \\ \text{mass of species A} \end{matrix} = \begin{matrix} \text{net rate of inflow} \\ \text{of species A} \end{matrix} + \begin{matrix} \text{net rate of reaction} \\ \text{of species A} \end{matrix}$$

Assuming no reaction, but **not** steady state we can write:

$$\frac{d}{dt}(C_A V) = K_L a V (C_A^* - C_A)$$

since this semi-batch process has no inflow of liquid ( $L_1=0$ ). If the total volume in the tank remains constant in time (which is typically true), we get:

$$\frac{dC_A}{dt} = K_L a (C_A^* - C_A)$$

Solving this differential equation subject to the condition that the initial concentration is given as  $C_{A_0}$ , yields:

$$\ln \frac{C_A^* - C_{A_0}}{C_A^* - C_A} = K_L a t$$

**OUTCOME:**

*Calculate the mass exchanged in a well-mixed contactor or the time necessary to achieve a particular exchange*