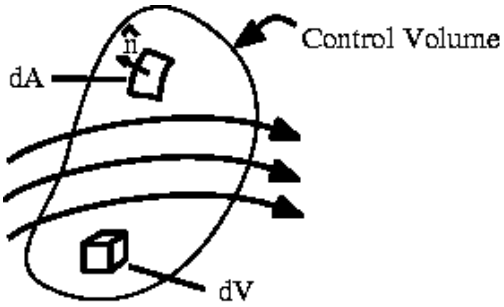
Shell/Integral Balances

Shell or integral (macroscopic) balances are often relatively simple to solve, both conceptually and mechanically, as only limited data is necessary. At the same time, many problems require only the level of detail that may be extracted from these types of balances and thus they represent powerful tools.

- Perform control volume (or macroscopic) balances
  1. Derive and use a macroscopic mass balance (continuity equation) [4.1, 4.2] [notes]
  2. Derive and use a macroscopic momentum balance [5.1, 5.2] [notes]
  3. Derive and use a macroscopic energy balance [6.1, 6.2] [notes]
- Use the General Mechanical Energy Balance Equation to perform calculations on simple pipe networks and/or process problems [6.3]
  1. Explain the connection between a macroscopic energy balance and Bernoulli's Equation
  2. Use Bernoulli's Equation to calculate velocities, pressures, or height changes in flowing fluid systems [notes]
  3. Extend the Bernoulli's Equation for more general cases (i.e., write and use the mechanical form of the macroscopic energy balance equation) [notes]
- Solve heat transfer problems on macroscopic bodies [p241-243]
  1. Determine the efficiency of a fin [notes]
  2. Calculate the heat flow through a fin from the fin's efficiency [notes]

SIB: Mass Balance

For our balance, we will choose an **arbitrary** fixed (in space) control volume. Thus, it is an *open* system (i.e., one through which mass may flow). Because of this, the total material within our control volume can change with time as stuff flows in and out.



A "word equation" for this process may be written as:
rate of change of mass in CV = *net* rate of mass flow *into* CV

We note that the amount of mass contained in a small section of volume, dV, can be written as ρ dV. In order to calculate all of the material in our control volume we simply add all of the contributions from each section dV (i.e., we integrate), so that the rate of change of mass in the CV is:

$$\frac{d}{dt}\iiint \rho dV$$

The easiest way to capture the amount of material flowing into/out of the CV is to watch the stuff that crosses the boundaries. That is, if we look at the velocity of the mass and see what portion of that velocity is *perpendicular* to the surface, we can then add up all the contributions of stuff crossing the surfaces, so that our final equation can be written as:

$$\frac{d}{dt}\iiint \rho dV = -\iint \rho(v \cdot n) dA$$

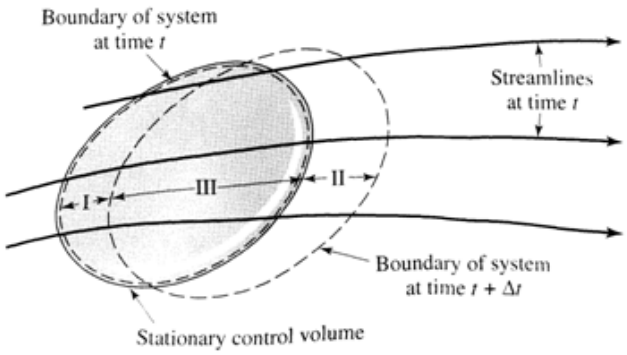
Functionally, what this equation means is that, in order to perform a macroscopic mass balance, we simply look at the inlets and outlets of a process and multiply the (area) average velocity times the density and the cross-sectional area of that orifice.

OUTCOME:

Derive and use a macroscopic mass balance (continuity equation)

SIB: Conservation of Momentum: Basic System Statement

If we now choose an **arbitrary** system of material (for a Lagrangian outlook on the problem):



we can write a "word equation" for the changes in momentum of this system as:

$$\begin{pmatrix}\text{rate of change}\\ \text{of momentum}\\ \text{of fluid system}\end{pmatrix} = \begin{pmatrix}\text{sum of the forces}\\ \text{acting on the}\\ \text{fluid system}\end{pmatrix}$$

Writing this mathematically we get:

$$\frac{d}{dt}\iiint_{SYS}\rho v dV = \sum F$$

where $\sum F$ denotes the sum of the forces acting on the fluid element within our system. In order to mathematically translate between a control system and a control volume one uses the *Reynolds Transport Theorem* that stipulates that:

$$\frac{d}{dt}\iiint_{SYS}\rho v dV = \frac{d}{dt}\iiint_{CV}\rho v dV + \iint_{CV}\rho v(v\cdot n)dA$$

so that our momentum balance equation can be written as:

$$\frac{d}{dt}\iiint_{CV}\rho v dV = -\iint_{CV}\rho v(v\cdot n)dA + \sum F$$

NOTE:

It was easier to rationale the balance of momentum using a system -- since we are all familiar with the equation $F = ma = m\frac{dv}{dt}$*. It might have also helped when we did a material balance, as a material balance on a system would simply have been* $\frac{d}{dt}\iiint_{SYS}\rho dV = 0$

Recalling from earlier in the course, we know that the potential forces that act on fluids are the following:

- Pressure force at inlets and outlets (a surface force): $F_P = -\iint_{inlet}^{outlet} PndA$
- Pressure and shear force exerted by wall surfaces on fluid in CV (surface forces): $R \equiv -\iint_{walls} PndA + \iint_{walls}\tau dA$

- Gravitational force exerted on all fluid within CV (a body or volumetric force): $W \equiv \iiint_{CV} \rho g dV = \rho V g$

Combining these we can write a general form of the momentum balance equation as:

$$\frac{d}{dt}\iiint_{CV} \rho v dV = -\iint_{CV} \rho v (v \cdot n) dA - \iint_{CV} P n dA + \iint_{CV} \tau dA + \iiint_{CV} \rho g dV$$

NOTE:

Recall that the shear stresses affect only the walls of the CV; the pressure affects all surfaces (including inlets and outlets); and the gravitational forces affect the entire CV.

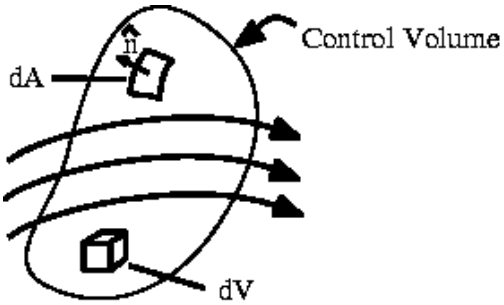
IMPORTANT:

This is a vector equation so that, in practice, there will be three momentum balances equations to write; one in each of the coordinate directions!

OUTCOME:

Derive and use a macroscopic momentum balance.

SIB: Energy Balance



For our balance, we will choose an **arbitrary** fixed (in space) control volume. Thus, it is an *open* system (i.e., one through which mass may flow). Accounting for energy within that volume (and transfer with surroundings) we get:

rate of accumulation of energy in CV	=	*net* rate of energy transport *into* CV (by flow)	+	*net* rate of energy transferred *to* material in CV

We denote *e* as the total energy per unit mass, and note that energy can be transferred to the material in the CV through both heat (thermal energy) flow and work. Therefore, we can write this in equation form as:

$$\frac{d}{dt}\iiint_{CV} e\rho dV = -\iint_{CS} e\rho(v\cdot n)dA + q + W$$

NOTATION:

- e, the total energy per unit mass, is comprised of mechanical (kinetic and potential) as well as internal (thermal) energy

- q represents the heat that is transferred to the material within the volume

- W represents any work done on or by the system (the chosen sign convention is that work done on the system is positive).

We note that the energy per unit mass may have an internal energy component as well as a kinetic and potential energy component, so that we can write it as:

$$e = \hat{U} + \frac{1}{2}v^2 + gz$$

We further note that the work can have several components:

$$W = W_s + W_\sigma + W_\tau$$

where W_S is the shaft work and the other two components (W_σ and W_τ) correspond to stresses at the boundaries (both normal and shear). If we extract the pressure from the normal stresses we can write our work in the form:

$$W = W_s - \iint_{CS} P(v\cdot n)dA + W_\mu$$

where the pressure stresses now make up what is typically called the "flow work" term, and remaining viscous stresses are lumped into the term W_μ, which is only non-zero if we have a non-zero shearing velocity on the control surfaces.

Combining these observations into a final form of the macroscopic energy balance equation, we get

$$\frac{d}{dt}\iiint_{CV}\rho\left(\hat{U}+\frac{1}{2}v^2+gz\right)dV = -\iint_{CS}\rho\left(\hat{U}+\frac{1}{2}v^2+gz+\frac{P}{\rho}\right)(v\cdot n)dA+q+W_s+W_\mu$$

OUTCOME:

Derive and use a macroscopic energy balance

SIB: Bernoulli Equation

A very special form of the energy balance equation will arise under certain limiting assumptions.

DEFINITIONS:

*The **Bernoulli equation** is obtained from the energy balance equation when we have a steady, incompressible, and inviscid flow in which there is no shaft work, no heat transfer, and no change in internal energy occurs.*

DEFINITIONS:

*An **inviscid flow** is one in which viscous losses are negligible (i.e., the fluid viscosity is very small).*

Let's look at each of these limitations in turn:
Steady Flow: the left-hand side becomes zero as the d/dt term vanishes
Incompressible Fluid: the density remains constant everywhere
Inviscid Flow: there are no viscous work losses ($W_\mu = 0$) **and** the flow is everywhere a "plug flow" (i.e., velocities are constant perpendicular to the main flow -- this *also* means that the shear stresses are zero!)
No Heat Transfer (adiabatic): q=0
No Change in Internal Energy: $\hat{U}$ drops out of the equation
Under these simplifications our macroscopic energy balance becomes:

$$0 = -\iint_{CS} \rho\left(\frac{1}{2}v^2 + gz + \frac{P}{\rho}\right)(v \cdot n)dA$$

Because of our constant velocities and constant densities, we can simplify this further to be

$$\left(\frac{1}{2}v^2 + gz + \frac{P}{\rho}\right) = constant$$

NOTE:

The Bernoulli Equation (sometimes called Bernoulli's Law) is typically used because it is true along a streamline (i.e., for a CV that encircles a single streamline), although it will clearly also work for more complex CVs that satisfy the above criteria.

OUTCOME:

Explain the connection between a macroscopic energy balance and Bernoulli's Equation

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SIB: Mechanical Energy Balance Equation

Perhaps the most common way to write the Bernoulli equation is:.

$$\left(\frac{1}{2}\rho\,\Delta\,v^2+\rho g\,\Delta\,z+\,\Delta\,P\right)=0$$

If you allow for frictional losses in the Bernoulli equation, then this equation becomes:

$$\left(\frac{1}{2}\rho\,\Delta\,(v^2)_{avg}+\rho g\,\Delta\,z+\,\Delta\,P\right)=\rho g h_L$$

NOTE:

When you include frictional losses it is no longer correct to assume that you have a plug flow, so you need to use the average value of the v^2 term at each point.

where $\rho g h_L$ represents the *frictional losses* in the system.

NOTE:

Recall that we learned to calculate the h_L calues earlier in the term. One last tweak to the Mechanical Energy Balance equation is to note the following:

$$(v_{avg})^2 \neq (v^2)_{avg}$$

To allow the simplified statement above to be most conveniently used, one will often define a value α as $\alpha=\frac{(v^2)_{avg}}{v_{avg}^2}$ so that you may write:

$$\left(\frac{1}{2}\rho\,\Delta\,\alpha(v_{avg})^2+\rho g\,\Delta\,z+\,\Delta\,P\right)=\rho g h_L$$

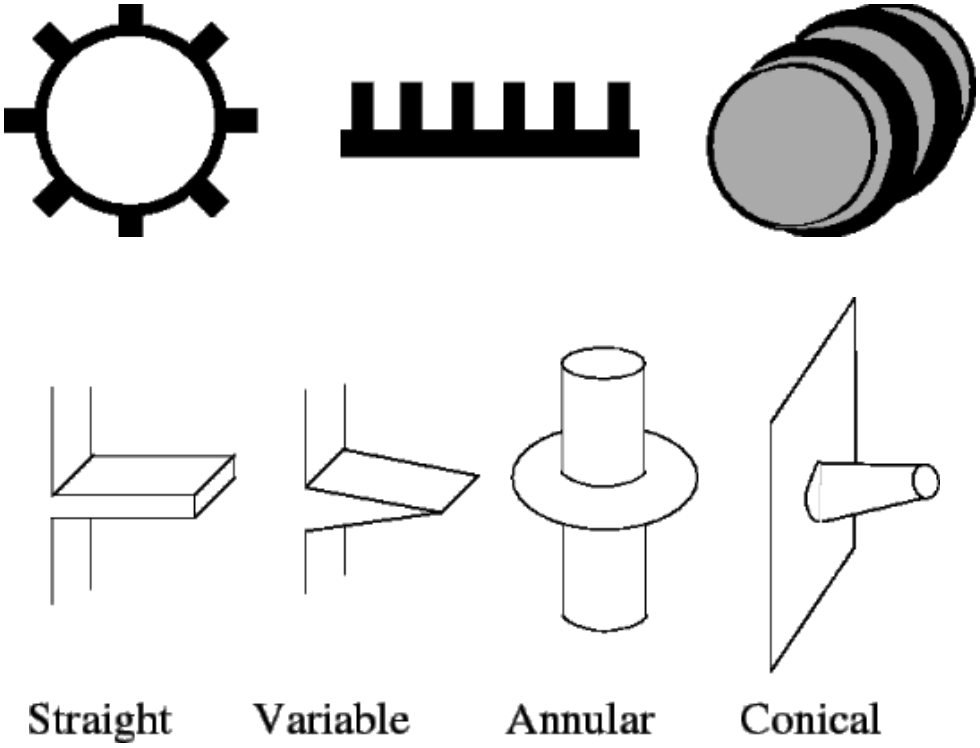
OUTCOME:

Extend the Bernoulli's Equation for more general cases (i.e., write and use the mechanical form of the macroscopic energy balance equation)

SIB: Fins

A few lectures ago we looked at how for the case of a cylinder we could, in fact, *increase* the rate of heat transfer by adding insulation (i.e., making the diameter larger). Upon reflection, we realized that this happens because by adding the insulation (or making the diameter larger) we *increased the area available for convective heat transfer*! When we figured that out it seemed that everyone was happy with what was a confusing issue. If we had instead resorted to thinking about finned surfaces (like the radiator in your car, the little do-dads in your computer, etc.) we may have realized earlier that sometimes adding material *increases* heat transfer (as long as the increase in surface area is sufficient to cause the convection to overcome the added conductive resistance). This will be our topic for a short while.

Having said that, I am sure that it is perfectly clear that often, when we need to increase the rate of heat transfer through a material, we may add *fins*! Some typical fin configurations are shown below.



As I am sure that you can imagine, fins are tremendously technologically important items (as Chem. E's we would certainly be interested in their use in heat exchangers!). So, how do we handle these things?

SIB: Using the Fin Efficiency

The more practical use of the fin efficiency lies in the fact that, once it is calculated, the resulting *dimensionless* expression can be used to easily determine the actual heat flow from any similar fin.
In other words, we can rearrange the fin efficiency definition to yield:

$$q_{fin} = \eta_f h A_f (T_{BASE} - T_{bulk})$$

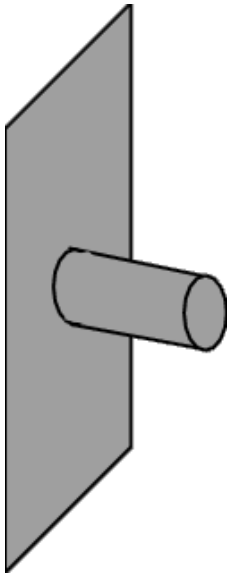
so that simply "looking up" the efficiency value η_f yields our solution.

OUTCOME:

Calculate the heat flow through a fin from the fin's efficiency

TEST YOURSELF

Let's look at an example...



You are evaluating the introduction of "pin"-type fins onto the surface of your company's new heat exchanger. You suggest that, in order to compare the two scenario's realistically, you need to calculate the heat flow both with and without the fin. As long as you get at least a 10% increase in heat flow, you will go ahead and do it.

Should you do it? (Your proposal is to put 25 aluminum (k=200W/mK) pins that are 3 cm long fins of 1 cm diameter on every 100 cm^2 of surface. Your heat transfer coefficient is 10 W/m^2K.)
Since the heat flow off the unfinned surface and the fins themselves are in **parallel**, we simply add them to get the total heat flow, so

$$q = q_{unfinned} + q_{fin} = hA_o \Delta T + \eta_f h A_f \Delta T = h(A_o + \eta_f A_f) \Delta T$$

where A_o is the area of the unfinned surface.

NOTE:

We can therefore think of the fin efficiency as a correction factor for the amount of the surface area of the fin that is actually "useful" (as useful as the unfinned surface, that is).

One final important issue to note is that the simple expression for fin heat flow,

$$q_{fin} = \eta_f h A_f \Delta T$$

lends itself quite easily to our resistor analysis, to yield

$$R_{fin} = \frac{1}{\eta_f h A_f}$$

NOTE:

*Be careful to remember that the fin heat flow (and therefore the fin resistance) accounts for **both** conduction and convection through/ off the fin.*
