Engineering Calculations

In order to properly communicate with other engineers / scientists you must use the correct *language* .How can you use concise language to convey not only what you measured, but how precise the measurement was ?

- Explain the utility of the proper use of units
  - # Define and give examples of value, units, and dimensions in a given expression
  - # Convert from one set of units to another
  - # Identify an invalid equation, based on dimensional arguments
  - # Compare two quantities using a dimensionless group
- Correctly use scientific notation
  - # Write a value in scientific notation(using the correct number of significant digits)
  - # Determine the number of significant digits both in a given value as well as in the result of arithmetic

While measuring data is enormously useful you cannot possibly measure every possible bit of information under every possible condition(it would take too long and would be far too expensive).How can you use limited data to obtain *enough* information to get by ?

- Utilize curve - fitting or linear interpolation to estimate unmeasured data from measured data
- "Check" your results by order - of - magnitude estimation, back substitution, etc.

EC: Define and give examples of value, units, and dimensions in a given expression

Using units properly

You use values, units and dimensions all the time:

Grocery List

- 1 carton of milk (value: 1; units: carton; dimensions: volume ($length^3$))
- 1/4 pound burger (value: 0.25; units: pounds; dimensions:mass)

DEFINITION:

*A **value** is the numerical quantity. For example: 5.2*

DEFINITION:

*The **units** tell what that quantity represents. For example: 5.2 liters.*

DEFINITION:

*The **dimensions** are the measurable properties that the units represent. For example: a liter is a unit of volume (units are a specific example of a dimensional quantity).*

OUTCOME:

Define and give examples of value, units, and dimensions in a given expression

TEST YOURSELF!

Give some more examples of value, unit, dimension triplets.

In scientific apps. -> units play a central role in quantification!

Treat units as algebraic entities:

- add/subtract them, if they are of the same units
- multiply/divide them anytime

EXAMPLE:

$$(x)\left(\frac{y}{x}\right) = y$$

Algebraic equation

$$(dozen\ eggs)\left(\frac{dollars}{dozen\ eggs}\right) = dollars$$

"Unit" equation

$$1\,piggie + 2\,piggie = 3\,piggie$$

"Unit" equation

Converting units

System of units: SI units

Units are quite helpful:

What does "three rivers stadium is 4 away" mean? 4 **what**!?

Standardized units are **more** helpful:

"Three rivers stadium is 4 **miles** away" is more helpful than "three rivers is 15 lengths of my backyard away"

SI units are an international standard group of units based on the meter (length), second (time), kilogram (mass), degree Kelvin (temperature), ampere (current), lumen (light intensity), and mole (number of elementary entities).

NOTE:

The dimensions of each of the SI units is shown in parenthesis.

Other conventions: English units, CGS units

We will (primarily) use SI, but be familiar with others...

What units to use?

Use **any** that you like (but be consistent!).

You can (easily) convert them anyway! (see above)

Unit Conversions

Units with same dimensions -> easily inter-converted

EXAMPLE:

$$15\ steps = 25\ feet \ \rightarrow \ \frac{15\ steps}{25\ feet} = 1$$

OUTCOME:

Convert from one set of units to another

TEST YOURSELF!

So, if it is 1000 feet between Benedum and the Cathedral

$$1000\ feet\left(\frac{15\ steps}{25\ feet}\right) = 600\ steps$$

If there are 3.2808 feet in a meter, how many meters is one step?

Multiple and compound units

How old are you? 600,000,000 seconds!? Saying 19 years is a lot easier or you could just say 600 Megaseconds.

DEFINITION:

Multiple units are units devised purely for convenience. They can be of two types: new names (1 hour is 3600 seconds), or prefixed units (1000 millimeters is 1 meter).

NOTE:

The dimensions of multiple units must be the **same** as the base unit (hours and seconds and days, etc. are all **time** units).

What units do you use for velocity, volume, pressure?

Treat units algebraicly -> moving 1 mile every 2 hours means:

$\left(\frac{1\,mile}{2\,hours}\right) = 0.5\,miles/hour$

Your container is 10 cm wide, 10 cm long, and 10 cm high. The volume is:

$(10cm)(10cm)(10cm) = 1000cm^3$

1000 cm^3 is often called a liter (for convenience).

EXAMPLE:

You may recall from physics:

$v(m/s) = v_o(m/s) + a(m/s^2)t(s)$

Do these units match?

DEFINITION:

Compound units are units made up through algabraic combination of base units. They are often given special names for convenience (volume: liter; force: Newton (pound); pressure: Pascal).

OUTCOME:

Convert from one set of units to another

TEST YOURSELF!

*Can you convert **compound** units? Let's give it a shot!*

EC: Identify an invalid equation, based on dimensional arguments

Dimensional Equations

Dimensions play an important role -> equation validity!

It doesn't make sense to say that: 1 meter = 25 seconds!

Conversely, we already said that 3.2808 feet = 1 meter is valid. Why? [hint]

Dimensional Homogeneity

The dimensions on both sides of the equals sign must be the same for an equation to be valid. Another way to say this is that valid equations must be dimensionally homogeneous.

It is a good practice to make **units** the same (via conversion).

It is also good practice (and a course requirement!) to show all units throughout a problem to test equation validity!

OUTCOME:

Identify an invalid equation, based on dimensional arguments

TEST YOURSELF!

Which of these equations is dimensionally homogeneous?

$$x(m) = x_o(m) + 0.3048(m/ft)v(ft/s)t(s) + \tfrac{1}{2}a(m/s^2)[t(s)]^2$$

$$P\left(\frac{kg}{ms^2}\right) = 101325.0\left(\frac{Pa}{atm}\right)1\left(\frac{\frac{kg}{ms^2}}{Pa}\right)P_o(atm) + \rho(kg/m^3)v(m/s)$$

NOTE:

It is sometimes simpler to write the units as "vertical fractions" to facilitate canceling.

IMPORTANT:

Just because an equation is dimensionally homogeneous does **not** mean that it is valid! Dimensional considerations act as a first test for validity only!

Dimensionless numbers

Pure numbers have no dimension: 1, 5.2, 3.14159
(sometimes **constants** do, however!)

Alternately, sometime all units cancel out: 6 ft/1ft = 6

DEFINITION:

> ***Dimensionless quantities*** *can be either pure numbers, or dimensionless* ***groups*** *(expressions where all the units cancel).*

What are dimensionless groups good for?

For one thing, dimensionless groups are useful for "pure" comparisons.

OUTCOME:

> Compare two quantities using a dimensionless group

TEST YOURSELF!

> *Which is further away, Cleveland (136 miles) or State College (223 km)?*

Must match units in order to compare (in miles or km?).

NOTE:

> Dimensionless groups can be **any** expression whose units cancel! Here we consider ratio of lengths and times, but force, pressure, resistance, etc. all work as well! The only thing you **can't** have is a ratio of things with different dimensions! (Obviously, since they would no longer cancel.)

Scientific Notation

Very large and very small numbers are cumbersome.

EXAMPLE:

one option: 600,000,000s = 600 Ms
another: 600,000,000s = $6x10^8$s

Either option (compound units or scientific notation) works for both large and small (0.000001m = 1 micrometer = $1x10^{-6}$m)

OUTCOME:

Write a value in scientific notation (using the correct number of significant digits)

TEST YOURSELF!

Write the number of seconds in a day using scientific notation.

Why use scientific notation?

Significant Figures

Represent the precision at which a value is known.

EXAMPLE:

Clock without a second hand -> 10:21 AM
Clock with a second hand -> 10:21:08 AM
Stop watch -> 10:21:08.12 AM

Writing numbers to show sig. figs.

- with decimal: count from non-zero on left to last digit on right
- without decimal: count from non-zero on left to last non-zero on right
- scientific notation: typically only sig. figs. are shown

EXAMPLE:

40500 ($4.05x10^4$) -> 3 sig. figs.
40500. ($4.0500x10^4$) -> 5 sig. figs.
0.0012 ($1.2x10^{-3}$) -> 2 sig. figs.
0.001200 ($1.200x10^{-3}$) -> 4 sig. figs.

Arithmatic Rules

It is not good enough to know (and convey) the precision of *measurements*. We will almost always have to *do something* with those numbers, we therefore need to consider how is our precision affected by arithmatic manipulation.

- multiply and divide -> get lowest number of sig. figs.
- add or subtract -> the last sig. fig. in each number is compared, keep only the (last) sig. fig. which is furthest left (the "largest")

OUTCOME:

Determine the number of significant digits both in a given value as well as in the result of arithmetic

TEST YOURSELF!

How many significant digits are there in $2.04x10^{-3}$?

TEST YOURSELF!

Your Car weighs 2100 lb_f
You put an 8 lb_f bag in the trunk
What is the car's weight now? [cheat]

TEST YOURSELF!

You measure a samples mass to be 3kg
Calculate the samples weight (multiply by gravity: 9.81 m/s^2)
What is your answer? [cheat]

Data Manipulation

Fitting Data

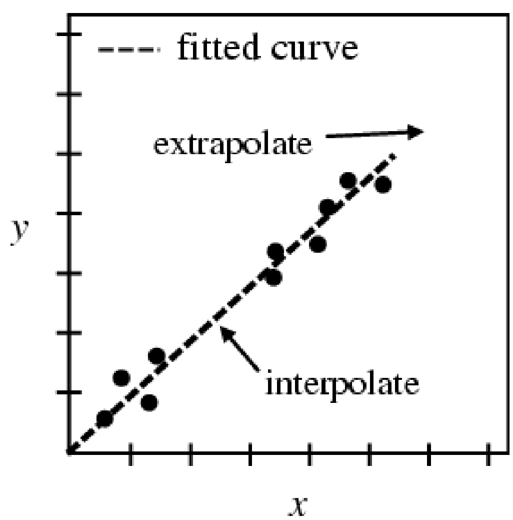
Want process variable -> can't directly measure it?

DEFINITION:

> ***Calibration*** *involves establishing a relationship between the process variable of interest and the measured quantity*

Once have C=f(x) -> plot data. *How do you use it?*

- Interpolation - get data **between** existing points.
- Extrapolation - get data **outside** existing points.
- Fit curve.



Interpolate:

$$y = y_1 + \frac{x-x_1}{x_2-x_1}(y_2 - y_1),$$

where x and y are the to-be-determined variables and x_i and y_i are the data on the curve.

Need to do more often (or extrapolate) -> develop an expression y=f(x)

Curve Fitting

Straight line -> easy

- by inspection ("looks good")
- least squares analysis -> minimize the sum of (absolute error)^2

Eqn. of line -> $y = ax + b$

a - slope -> $\frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$ b - intercept -> $b = y_1 - ax_1 = y_2 - ax_2$

Non-linear fitting -> cast non-linear equation [like $y=Ae^{bx}$] into linear form [like ($ln(y) = bx + ln(a)$].

Plot one y-dependent quantity [here, ln(y)] versus an x-dependent quantity [here, x].

OUTCOME:

Utilize curve-fitting or linear interpolation to estimate unmeasured data from measured data

TEST YOURSELF!

How do you "linearize": $y=Bx^3$ [cheat]

These two types occur often:

- semi-log (log-linear or linear-log): useful for exponential dependencies
- log-log: useful for polynomial dependencies

IMPORTANT:

Do *not* perform a "double log", that is, don't plot the ln(y) on log paper.

EC: "Check" your results by order-of-magnitude estimation, back substitution, etc.

How do I know I am right?!

check the numbers

- back-substitution (recalculate using answer)
- order-of-magnitude (approximate the math and do it in your head - are you within a factor of 10?)
- reasonableness - did you get a negative time or mass? a volume bigger than the ocean?

check the units

- is the equation dimensionally valid?
- does you answer have the correct dimension for what you **expected** to calculate?

OUTCOME:

"Check" your results by order-of-magnitude estimation, back substitution, etc.

TEST YOURSELF!

If I told you there were 310 million seconds in a year, how would you know if I was telling the truth? (Without using a calculator.)
